AY250 Assignment 5

due: Thursday, Nov 4, 2010

1 - In Sections 12.4 and 12.5, we used simple arguments to infer the fate of clouds following mass loss that is fast or slow with respect to the dynamical time. In the context of star formation, it is common to recast these results in terms of an efficiency ϵ, the fraction of the original cloud mass that becomes stars.

(a) Suppose a cloud produces stars slowly. As these stars are born, they drive off a portion of the cloud mass through their winds, until the final system consists of all stars. As a function of ϵ, find the final-to-initial ratios of radius, velocity dispersion, and mean density: R_f/R_i, V_f/V_i, and ρ_f/ρ_i. Here, the initial values refer to the starless cloud, while final values pertain to the star cluster. As a specific example, determine these three ratios for $\epsilon = 0.3$.

(b) Now suppose the cloud produces its stars quickly, and that these stars instantaneously blow off the ambient cloud gas. Find the above three ratios, again as functions of ϵ. What are the ratios for $\epsilon = 0.6$?

(c) The observed velocity dispersion of the Pleiades is about 0.5 km s^{-1}. In contrast, a typical velocity dispersion in a dark cloud is 1.5 km s^{-1}. If the Pleiades formed rapidly, what was ϵ, according to this model? What must ϵ have been if the cluster formed slowly? How do these theoretical values compare with the *observed* star formation efficiency in regions like Taurus?

2 - Let us explore a simple model for the HH 34 giant jet. Here, the speed of Herbig-Haro knots declines linearly with distance. Thus, we can write for the jet velocity

$$\frac{dV}{dr} = -\frac{1}{t_\circ} ,$$

where r is the distance of the knot from the embedded star and $t_\circ$ is a constant with dimensions of time.

(a) Suppose a knot leaves the central star at $t' = 0$ with initial speed $V_\circ$. After it has traveled a time t', what is its speed V?

(b) What is r_∞, the radius the knot asymptotically reaches as t' becomes infinite?

Suppose the jet source is rotating with angular speed ω as it emits knots into a plane. After each knot leaves, it continues to travel outward along a straight radius. Thus, at any time t, a knot located at angle θ has been traveling for a time

$$t' = t - \theta/\omega .$$

(c) What is $r(t, \theta)$, the radial distance of a knot with angle θ at time t? Your expression should involve r_∞, ω, and $t_\circ$.

Define $r' \equiv r/r_\infty$, $\gamma \equiv 1/(\omega\, t_\circ)$, and $\theta' \equiv \theta - \omega\, t$. The nondimensional function $r'(\theta')$, plotted in plane polar coordinates, is the shape of the curve made by all the knots. This function contains the single parameter γ. Note that θ' is negative; it varies from -∞, for a source emitted in the remote past, to 0, for a source emitted recently.

(d) By plotting $r'(\theta')$ for various γ-values, find one that gives an acceptable fit to the HH 34 jet, as seen in Figure 13.7. Ignore any distortion in the observed curve due to projection. Note that your curve represents one arm of the jet; the other is obtained by rotation through 180°.

(e) For the HH 34 jet, $V_\circ$ is observed to be 490 km s^{-1}, and $r_\infty = 1.8$ pc. What is $T_\circ$, the rotation period of the jet source in years?

3 - As we noted in §13.4, the idea that disks generate winds faces the difficulty that the same inward flow would spin up the star to unacceptably high rotation rates. This problem can be avoided if the star, while accepting matter from the disk, still generates its own wind. Using a simple model, the mass *outflow* in the wind is then proportional to the mass *inflow* from the disk.

Let the accreting object be a low-mass, fully convective protostar, rigidly rotating at rate Ω_*. This rate, we posit, is a fixed fraction of the breakup value:

$$\Omega_* = f \sqrt{\frac{G\, M_*}{R_*^3}} \,,$$

where $f \ll 1$. From Figure 11.6, it is approximately true that R_* is proportional to M_*, as a result of the deuterium thermostat.

(a) Show that, under these assumptions and approximations, the star's equatorial velocity $V_{\rm eq}$ is a constant, independent of mass.

(b) The stellar angular momentum is given by

$$J_* = \beta\, M_*\, R_*^2\, \Omega_* \,,$$

where β is another constant (about 0.14). Express J_* as a function of M_* and $V_{\rm eq}$, times the appropriate coefficients.

(c) Suppose that matter flowing in from the disk has somehow been forced into corotation with the star. If $\dot{M}_{\rm in}$ is the mass inflow rate, what is $\dot{J}_{\rm in}$, the rate at which angular momentum is being advected onto the star?

(d) Finally, let the specific angular momentum being carried outward by the wind be $\gamma\, \Omega_*\, R_*^2$, where the order-unity constant γ must be determined from wind theory. Let

$\dot{M}_{\rm out}$ be the mass efflux in this wind. Demand that $\dot{M}_{\rm in} - \dot{M}_{\rm out}$ be the rate of stellar mass increase $\dot{M}_*$. Also demand that $\dot{J}_{\rm in} - \dot{J}_{\rm out}$ be the rate of stellar angular momentum increase $\dot{J}_*$, where J_* was determined in (b). Find the relation between $\dot{M}_{\rm out}$ and $\dot{M}_{\rm in}$. What are the limits on γ such that $\dot{M}_{\rm in} > \dot{M}_{\rm out}$?

4 - An interesting question is whether the masers observed in star-forming regions are saturated or not. We may investigate the matter by deriving an expression for T_s, the brightness temperature of a maser just at the point of saturation.

(a) Equation (14.10) gives the critical mean intensity $\bar{J}_s$. After neglecting the collisional term in the numerator and setting $g_u = g_l$, obtain the corresponding specific intensity I_s.

(b) At the transition to saturation, $I_s = I$. Using equation (14.1) to relate I and T_s, find the desired expression for the latter quantity. It is simplest to cast T_s in terms of $\nu_\circ$, $A_{\rm ul}$, Γ, and the beaming solid angle $\Delta\Omega$.

(c) In a long, filamentary maser, we may set $\Delta\Omega \approx (d/s)^2$, where d is the filament diameter and s the path length over which the radiation is amplified. Consider an OH maser with $\nu_\circ = 1665$ MHz. Here, $A_{\rm ul} = 7.1 \times 10^{-11}$ s^{-1} and $\Gamma = 0.03$ s^{-1}. If we observe $T_B = 10^{13}$ K and $d = 10^{14}$ cm, what is the path length required for saturation? Can this length reasonably be attained?

(d) Now repeat this exercise for an H_2O maser with $\nu_\circ = 22$ GHz. Here, $A_{\rm ul} = 1.9 \times 10^{-9}$ s^{-1} and $\Gamma = 1.0$ s^{-1}. If $T_B = 10^{14}$ K and $d = 10^{13}$ cm, what is the required length now? Is this an astrophysically plausible distance?

5 - The question of maser saturation may also be addressed by considering the observed linewidths. As we have seen, the typical width of a 22 GHz H_2O maser in W49 is $\Delta V_r = 0.5$ km s^{-1}. A representative brightness temperature, from equation (14.2) is $T_B = 3 \times 10^{15}$ K.

(a) Suppose the radiation, on its way from the source to us, is unsaturated. Let $T_B(0)$, the background brightness temperature, be 3 K. What is the maser gain under these circumstances?

(b) Still assuming the maser to be unsaturated, what is $\Delta V_r(0)$, the linewidth at the source location?

(c) If this linewidth is created thermally, what is the temperature at the source? Is this value realistic? How will saturation of the maser alter the situation?

6 - Some post-main-sequence giant stars excite OH maser emission primarily in the 1612 MHz line. The observed spectrum consists of two peaks, separated in velocity by

20-50 km s^{-1}. This kind of profile arises naturally if the emission stems from a spherical gas shell, expanding away from the central star.

(a) The relatively thin shell, of mean radius R, is far enough from the star that it coasts at V_∞, the terminal value reached by the stellar wind. Consider a radial line extending from the star to the shell, and deviating by an angle θ from the line of sight. For each small angular increment $\Delta\theta$, maser emission is amplified over ΔL, the line segment within the shell subtended by $\Delta\theta$ and directed toward the observer. Write an expression for ΔL.

(b) To maintain the velocity coherence necessary for maser amplification, the line-of-sight velocity V_r must change by no more than the Doppler width $\Delta\nu_D$. Find an expression for the maximal $\Delta\theta$ that gives velocity coherence.

(c) Combining your answers from (a) and (b), find ΔL algebraically as a function of V_r. For the typical values $\Delta v_D = 0.5$ km s^{-1} and $V_\infty = 15$ km s^{-1}, plot $\Delta L/R$ as a function of V_r/V_∞. If we interpret V_r as the line-of-sight velocity relative to the star, this plot should mimic the observed spectrum of the source, provided the maser is *saturated.* Do you see why?

(d) According to the model, maser emisison near the velocities $\pm V_\infty$ should be seen as two spots directly in line with the star. More generally, the model predicts that emission at any V_r is located in a ring of radius $a\,(V_r)$. Derive an expression for $a\,(V_r)$.

(e) The giant star OH127.8-0.0, located at a distance of 3.3 kpc, exhibits the typical double-peaked spectrum. The two peaks are separated in velocity by 22 km s^{-1}. Emission with V_r near 8.5 km s^{-1} is observed to be in a ring with an angular diameter of $2''$. What is the physical radius R of the shell?