

AY250 Assignment 7

due: Thursday, Dec 2, 2010

1 - From observations of spectral veiling in a certain classical T Tauri star, one finds that the total luminosity in ultraviolet and optical excess continuum radiation is $L_{\text{cont}} = 0.18 L_{\odot}$. Modeling the emission as arising from a hot slab, the temperature of the slab required to produce the continuum emission is $T_{\text{cont}} = 1.3 \times 10^4$ K. We may interpret these results physically, under the assumption that the excess radiation arises from infall onto the stellar surface.

This particular star has an observed spectral type of K7, corresponding to an effective temperature of $T_{\text{eff}} = 4000$ K. Applying a bolometric correction to the extinction-corrected J -band flux, one finds a stellar luminosity of $L_* = 1.7 L_{\odot}$.

- (a) What is the mass infall rate \dot{M} onto the star?
- (b) What is V_{in} , the infall velocity just above the stellar surface? Assume that gas elements start at least several stellar radii from the surface. Compare your answer with the infall speeds inferred from Figure (17.12) for the star S Cra.
- (c) What fraction f of the star's total surface area is covered by accretion hot spots?

2 - In equation (17.21), we found that L_D , the luminosity impinging on a flat, circumstellar disk, is about 0.2 times the full stellar value. Let us redo the derivation more carefully, in order to obtain an exact result.

- (a) Derive the propagation factor f_{θ} quoted in equation (17.16).
- (b) Equation (17.18) gives the flux F_{rad} striking a point lying on the disk surface. Integrate F_{rad} , as given in the middle form of this equation, over the entire disk, thereby obtaining L_D . Take the inner disk radius to be R_* and denote the outer one as ϖ_D .
- (c) Find L_D/L_* in the limit that $\varpi_D \gg R_*$. Compare your answer with the more approximate equation (17.21).

3 - When discussing planet formation, we described how gas pressure within a dusty disk creates sub-Keplerian orbital speeds and consequently a vertical shear. This shear, in turn, drives turbulence that inhibits grain coagulation. Let us evaluate the departure from Keplerian velocity in a typical disk.

Suppose the disk surface density declines as a power law

$$\Sigma(\varpi) = \Sigma_o (\varpi/\varpi_o)^{-n} ,$$

where ϖ_o is the inner radius, and where $n > 0$. Similarly, assume a power-law temperature variation, with associated exponent q , as in equation (17.6).

(a) Suppose the disk is vertically isothermal at any radius ϖ , and that the local sound speed is a_T . Find an expression for the midplane pressure P_c in terms of $\Sigma(\varpi)$, a_T , and the Keplerian angular speed Ω_{Kep} .

(b) What is the radial variation of P_c ? Specifically, find the index α , where

$$\alpha \equiv \frac{\varpi}{P_c} \frac{dP_c}{d\varpi} .$$

Express α in terms of n and q .

(c) It is the finite α -value that causes the orbital speeds u_ϕ of fluid elements to be sub-Keplerian. Let this velocity difference be $\Delta V \equiv |u_\phi - V_{\text{Kep}}|$. Derive an expression for $\Delta V/V_{\text{Kep}}$ in terms of α , a_T^2 , and V_{Kep} itself.

(d) Evaluate $\Delta V/V_{\text{Kep}}$ numerically at the Earth's orbit. For this purpose, let $q = 1/2$ and $n = 3/2$, where the latter is the standard assumption for the minimum mass solar nebula. Finally, assume a gas temperature of $T = 300$ K

4 - Consider more carefully how runaway growth occurs within a disk of planetesimals orbiting a young star. Let ρ_s be the volumetric mass density of the swarm. Focus on a relatively large planetesimal of mass M and radius R moving with speed v with respect to the other objects. If this planetesimal is on a circular orbit, then v is the velocity dispersion of the swarm.

(a) Find \dot{M} , the rate at which M increases by colliding with other members of the swarm. Note that the collision cross section is not just πR^2 , but is enhanced by gravitational focusing. The enhancement factor depends on the *Safronov number*, $\theta \equiv (1/2) (V_{\text{esc}}/v)^2$, where the escape speed $V_{\text{esc}} \equiv \sqrt{2GM/R}$. You may derive the enhancement factor (as you already did in Problem 3.3) by considering energy and angular momentum conservation.

(b) Over time, the density ρ_s changes along with h , the local scale height of the disk. The latter is the distance over which the density falls by $1/e$ from its midplane value. Assuming the velocity dispersion v is isotropic, find h in terms of v and the local circular velocity Ω .

(c) The product $\rho_s h \equiv \Sigma$, which is the surface mass density, may be considered fixed. Rewrite your expression for \dot{M} in terms of R , Σ , Ω , and θ .

(d) Let ρ_p be the internal density of the planetesimal, also a fixed quantity. From your answer for (c), find the growth rate \dot{R} . Verify that, if θ is held fixed, as in the traditional Safronov theory, $R(t)$ grows at a constant rate. This is *not* a runaway situation.

(e) Suppose, however, that $\theta \gg 1$ and that v is constant in time. Consider two growing planetesimals with radii $R_1(t)$ and $R_2(t)$. Show that, when R_2 becomes a certain function of the initial values $R_1(0)$ and $R_2(0)$, R_1 runs away to infinity

5 - Nonhomologous contraction occurs when a star's luminosity profile, $L_{\text{int}}(M_r)$, peaks internally, as exemplified in the $t = 0$ curve of Figure 18.7b. Here, you will calculate a generic luminosity profile yourself.

Assume that L_{int} is L_{rad} , as given in equation (11.29). Take the opacity κ to obey Kramers law (see Appendix G). Assume, moreover, that the star consists of a monatomic, ideal gas, in which the specific entropy is a spatial constant.

(a) Show that L_{int} is a dimensional constant L_1 times $m p^{7/5}$, where $m \equiv M_r/M_*$ and $p \equiv P/P_o$. Here, M_* is the star's total mass and P_o its central pressure.

(b) We next need the dependence of p on m . We proceed indirectly, first finding the spatial distribution of the density. Equations (10.26) and (11.14) govern the variation of $M_r(r)$ and $P(r)$, respectively. Define a nondimensional density $\phi \equiv (\rho/\rho_o)^{2/3}$, where ρ_o is the central value. Similarly, define a nondimensional radius $\xi \equiv r/r_o$, where

$$r_o \equiv \left(\frac{5 P_o}{8 \pi G \rho_o^2} \right)^{1/2} .$$

By combining equations (10.26) and (11.14), find a nondimensional, second-order differential equation for $\phi(\xi)$.

(c) Integrate this equation numerically, subject to the appropriate boundary conditions at the star's center. Plot the function $\phi(\xi)$.

(d) The function $m(\xi)$ may be found from the nondimensional version of equation (10.26). This nondimensional equation contains a coefficient that is a combination of M_* , ρ_o , and r_o . By integrating analytically from the star's center to its edge, evaluate this coefficient.

(e) Now integrate numerically the equation for $m(\xi)$ and plot the result.

(f) After finding an expression for $p(\phi)$, plot L_{int}/L_1 as a function of m . This is the desired luminosity profile.

6 - Let us see, in a quantitative way, why CNO burning in intermediate-mass stars drives central convection. The energy generation rate from this process is

$$\epsilon_{\text{CNO}} = 1.33 \times 10^{27} \text{ erg gm}^{-1} \text{ s}^{-1} \left(\frac{\rho}{100 \text{ gm cm}^{-3}} \right) \left(\frac{T}{10^7 \text{ K}} \right)^{-2/3} \exp(-\theta) ,$$

where

$$\theta \equiv 70.7 \left(\frac{T}{10^7 \text{ K}} \right)^{-1/3} .$$

(a) Consider two ZAMS stars of 2 and 5 M_{\odot} . Their central temperatures are $\log T_c = 7.30$ and 7.42, respectively. The corresponding central densities are $\log \rho_c = 1.8$ and 1.2, where ρ_c is in gm cm^{-3} . Find ϵ_{CNO} numerically at the center of these two objects.

(b) Central convection occur when the energy generation rate is sufficiently high, i.e., when $\epsilon_{\text{CNO}} > \epsilon_{\text{crit}}$. Find an expression for ϵ_{crit} from L_{crit} in equation (11.29). Within L_{crit} itself, you may evaluate $(\partial T / \partial P)_S$ by assuming an ideal gas.

(c) Now evaluate ϵ_{crit} numerically for the above two stars. For the opacity, use the *maximum* of the appropriate Kramers law expression

$$\kappa = 2.0 \text{ cm}^2 \text{ gm}^{-1} \left(\frac{\rho}{100 \text{ gm cm}^{-3}} \right) \left(\frac{T}{10^7 \text{ K}} \right)^{-7/2} ,$$

and the electron scattering opacity:

$$\kappa = 0.34 \text{ cm}^2 \text{ gm}^{-1} .$$

Do both stars have central convection zones?