AY250 Final Exam - Problem 1

a)

A star's average internal temperature is proportional to its mass and inversely proportional to its radius (see, for instance, equation 11.2a):

$$\langle T \rangle \propto \frac{M_*}{R_*} \; .$$

In this problem, both stars have the same mass, so the difference in their internal temperatures is due only to their differing radii. Star B is half the size of star A, so it is twice as hot:

$$\langle T_B \rangle = 2 \langle T_A \rangle \; .$$

b)

The luminosity of a star, however, depends on its *effective* temperature T_{eff}, not $\langle T \rangle$. For stars on the convective part of their pre-main sequence track, T_{eff} is roughly constant and depends only on the stellar mass. Our stars, therefore, have the *same* effective temperature, which means that the difference in their luminosities is due only to their different sizes:

$$L \propto R^2 \; .$$

Since star A is twice the size of star B, it must have *four* times its luminosity:

$$L_A = 4 L_B \; .$$

The net flow of energy is therefore from A to B - paradoxically, since B has the higher internal temperature.

c)

As star A pours energy into star B, the virial theorem tells us that its gravitational energy will become *more negative*, and therefore its radius must shrink. By the same reasoning, star B will grow. This process will continue until the stars both reach the same radius R_f. At that point, their luminosities will be equal and there will be no more energy flow.

d)

At equilibrium, star A will have lost an amount of energy ΔE, and star B will have gained the same amount. Using the fact that a star's total energy can be written

$$E = -C\frac{M_*^2}{R_*}$$

where C is some positive constant, we can express energy conservation as for each star as

$$-C\frac{M_*^2}{R_A} - \Delta E = -C\frac{M_*^2}{R_f}$$

and

$$-C\frac{M_*^2}{R_B} + \Delta E = -C\frac{M_*^2}{R_f} .$$

Eliminating the ΔE from that pair of equations and using $R_A = 2R_B$, I find that

$$\begin{aligned} R_f &= \frac{2}{3}R_A . \\ &= \frac{4}{3}R_B . \end{aligned}$$

e)

For stars on the radiative portion of their main-sequence tracks, the internal temperature is still inversely proportional to R, so the answer to part A is the same. However, we can no longer compute the luminosity by assuming that the effective temperature is independent of radius. Instead, we must look to equation 16.14 in the book, which says that the critical luminosity that can be supported by radiation scales like $M_*^{11/2}R_*^{-1/2}$. For a fully radiative star, we can identify this with the *total* luminosity, and therefore conclude that

$$L_* \propto R_*^{-1/2} .$$

That means that *star B* now has the greater luminosity by a factor of $\sqrt{2}$. Hence, in the radiative regime, the energy flows from star B to star A.

By the same argument as before, star B will shrink as it loses energy and star A will expand. However, this time the result is that B's luminosity will get even higher and A's even lower. But that will tend to make B even smaller and A even bigger, and so on. Rather than equalizing the stellar radii as before, the process will then run away, so that $R_A \to \infty$ and $R_B \to 0$. In actual stars, this process would be halted by the onset of hydrogen burning in star B, which would fix ~~both radii at some final value~~. that star's radius. The radius of star A would continue to increase, from the energy supplied by hydrogen fusion in B.

Problem 2

A classical T Tauri star is observed to have $T_{\rm eff} = 3550$ K and $L_* = 1.0\,L_\odot$. The V-band flux from the star exhibits periodic variability, with a period of $P_* = 2.7$ days. Modeling of the star's spectral energy distribution shows it to be surrounded by a disk whose temperature varies with radius as

$$T(\varpi) = T_\circ\,(\varpi/\varpi_\circ)^{-q}\ .$$

Here, $T_\circ$ and $\varpi_\circ$ are the temperature and radius, respectively, of the inner disk edge. The numerical values are $T_\circ = 1500$ K, $\varpi_\circ = 6.0\,R_\odot$, and $q = 0.75$.

(a) What is the star's mass, M_*?

Both Figures 1.18 and 16.10 show the pre-main-sequence tracks for all relevant masses. For $\log\,(L_*/L_\odot) = 0.0$ and $\log T_{\rm eff} = 3.55$, we can see that $M_* = 0.4\,M_\odot$. We arrive at the same figure by interpolation of Table 16.2.

(b) Is the star rotating faster or slower than inner disk edge?

T Tauri disks are of such low mass that their rotation is Keplerian. Thus, the angular speed at the inner disk edge is

$$\begin{aligned}\Omega_\circ &= \sqrt{\frac{G\,M_*}{\varpi_\circ^3}}\\ &= 2.7\times 10^{-5}\ {\rm s}^{-1}\ ,\end{aligned}$$

where we have inserted the numerical values for M_* and $\varpi_\circ$. This rotation speed corresponds to a period

$$\begin{aligned}P_\circ &= \frac{2\,\pi}{\Omega_\circ}\\ &= 2.7\ {\rm days}\ .\end{aligned}$$

which matches the observed period of the V-band variation. Since the latter is caused by stellar rotation, the star is indeed corotating with the inner disk edge.

(c) What is the disk luminosity, L_D? Assume the emission comes from optically thick radiation by dust grains.

Each patch of the disk of temperature T emits a flux $\sigma_B\,T^4$ in either direction. We thus have

$$L_D/2 = 2\,\pi\int_{\varpi_\circ}^{\infty} d\varpi\,\varpi\,\sigma_B\,T^4\ .$$

We insert the given function form of $T(\varpi)$ and find that

$$
\begin{aligned}
L_D &= 4\,\pi\,\sigma_B\,T_\circ^4\,\varpi_\circ^{4q}\int_{\varpi_\circ}^{\infty} d\varpi\,\varpi\,\varpi^{-4q} \\
&= \frac{4\,\pi\,\varpi_\circ^2\,\sigma_B\,T_\circ^4}{4q\,-\,2} \\
&= 0.16\,L_\odot\ ,
\end{aligned}
$$

after also using the given values of q and $T_\circ$.