Arecibo Observatory Technical Memo AOTM 99-01

THE LBW FEED: THE MUELLER MATRIX AND ITS FREQUENCY DEPENDENCE [1]

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ABSTRACT

During February-March 1999 we made extensive observations to obtain a complete characterization of the polarization properties of the LBW feed over the frequency range 1420.4 ± 6.25 MHz in the form of its Mueller matrix. Generally speaking the Mueller matrix is nearly diagonal, which is the ideal; the feed is very well suited for polarization measurements of the 21-cm line. However, we were surprised to find that the feed exhibits a strong resonance and that there are some problems even outside this resonance. In an appendix, we also present a table of polarization calibrators useful at Arecibo.

The executive summary of this report is a succinct three sentences:

(1) Outside the resonances, the feed is very well suited for polarization measurements because its Mueller matrix is nearly diagonal. The system is stable enough that excellent accuracy can be obtained for all Stokes parameters, even for the difference between the two channels.

(2) The feed contains resonances that significantly affect polarization measurements. We found a ~ 1 MHz-wide resonance near 1424 MHz; in this resonance the Mueller matrix becomes severely non-diagonal and the feed is useless for accurate polarization measurements. Other observers have experienced resonances at different frequencies.

(3) Even outside the resonances, the polarized Stokes parameters Q, U, and V are nonzero. They add ~ 3.1 K of polarized power to the Stokes I blank-sky system temperature. There may be an additional unpolarized contribution, which we have not measured.

1. OUR SCHEME OF MUELLER MATRICES

The key to making accurate polarization measurements is calibration of the four Stokes parameters. The measured parameters S' are related to the true ones S by the Mueller matrix M:

$$\mathbf{S}' = \mathbf{M} \cdot \mathbf{S} \ . \tag{1}$$

[1] This revised version of 6 May 1999 supersedes an earlier version.

Here $\mathbf{S}$ is the usual 4-element Stokes parameter vector

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} ; \tag{2}$$

The Mueller matrix $\mathbf{M}$ represents the transfer function between true and measured Stokes parameters and is, in general, a 4×4 matrix in which all elements may be nonzero. In the usual way, we write

$$\mathbf{M} = \begin{bmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{bmatrix} . \tag{3}$$

The matrix elements are just the partial derivatives, for example

$$m_{IQ} = \left| \frac{\delta I}{\delta Q} \right|_{U,V} . \tag{4}$$

The observing system consists of several distinct elements. Each has its own Mueller matrix. Here we consider four, which we consider in sequence as seen by the incoming radiation. We the have azimuth-conversion matrix $\mathbf{M_{AZ}}$, the feed matrix $\mathbf{M_F}$, the amplifier matrix $\mathbf{M_A}$, and the backend matrix $\mathbf{M_B}$. The system Mueller matrix is the product of them all in the proper sequence: $\mathbf{M_{tot}} = \mathbf{M_B} \cdot \mathbf{M_A} \cdot \mathbf{M_F} \cdot \mathbf{M_{AZ}}$.

Two of these, $\mathbf{M_{AZ}}$ and $\mathbf{M_B}$, are trivial because they are only a matter of definition; we will discuss them later. The other two require a derivation, which in turn requires considering how the devices affect the amplitude and phase of the voltages in the two polarization channels. We represent the two complex voltages by a vector, and each system component has a voltage transfer function which is a 2×2 matrix. These are referred to as Jones vectors and matrices (Tinbergen 1996).

Each Jones matrix has its Mueller matrix counterpart. It is straightforward to relate the two types of matrix. First we must define how we measure and calculate the Stokes parameters from the voltages in the two polarization channels. For a dual-linear feed, the incoming sky radiation encounters a feed whose nominal outputs are voltages E_X and E_Y, where X and Y refer to the two directions of polarization. For this case we write the Stokes parameters in terms of the voltages as follows:

$$I = E_X \bar{E}_X + E_Y \bar{E}_Y \tag{5a}$$

$$Q = E_X \bar{E}_X - E_Y \bar{E}_Y \tag{5b}$$

$$U = E_X \bar{E}_Y + \bar{E}_X E_Y \tag{5c}$$

$$iV = E_X \bar{E}_Y - \bar{E}_X E_Y \tag{5d}$$

where $i = \sqrt{-1}$ and the bar over a symbol indicates the complex conjugate. If one has a dual-circular feed instead, one simply permutes the parameters (Q, U, V) to become (V, Q, U).

While discussing definitions we emphasize Stokes V. We follow the IEEE definition, namely

$$V = LCP - RCP \tag{6}$$

where LCP means left-hand-circular polarization. LCP is generated by transmitting with a left-handed helix, so from the transmitter the E vector appears to rotate anticlockwise. From the receiver, LCP appears to be rotating clockwise.

Astronomical sources don't exhibit much circular polarization so we must define the sense of V by radiating a calibration signal with a helical radiator, being sure to account for a reversal in sense upon each reflection. One sets $m_{VV} = \pm 1$ depending on the results.

1.1. The feed

Here we follow Stinebring (1982), who followed the procedure of Conway and Kronberg (1969):

$$\begin{bmatrix} E_{X,out} \\ E_{Y,out} \end{bmatrix} = \begin{bmatrix} 1 & \epsilon_1 e^{i\phi_1} \\ \epsilon_2 e^{-i\phi_2} & 1 \end{bmatrix} \begin{bmatrix} E_{X,in} \\ E_{Y,in} \end{bmatrix} . \tag{7}$$

Here the ϵ's represent undesirable cross coupling between the two polarizaions; for example, this might be caused by the two linear probes not being exactly 90° apart. The ϕ's are the phase angles of these coupled voltages. This equation assumes that the feed is "good", meaning that we need retain only first-order terms in the voltage gain (but we allow the phase to be arbitrary).

With the first-order approximation of equation 7 it is only a matter of a little algebra to calculate the feed's Mueller matrix (this could also be found by a suitable reduction and rearrangement of Stinebring's equation (A.5):

$$\mathbf{M_F} = \begin{bmatrix} 1 & 0 & \Sigma\epsilon\cos & \Sigma\epsilon\sin \\ 0 & 1 & \Delta\epsilon\cos & \Delta\epsilon\sin \\ \Sigma\epsilon\cos & -\Delta\epsilon\cos & 1 & 0 \\ \Sigma\epsilon\sin & -\Delta\epsilon\sin & 0 & 1 \end{bmatrix} , \tag{8}$$

where $\Sigma\epsilon\cos = \epsilon_1 \cos\phi_1 + \epsilon_2 \cos\phi_2$; $\Sigma\epsilon\sin = \epsilon_1 \sin\phi_1 + \epsilon_2 \sin\phi_2$; $\Delta\epsilon\cos = \epsilon_1 \cos\phi_1 - \epsilon_2 \cos\phi_2$; $\Delta\epsilon\sin = \epsilon_1 \sin\phi_1 - \epsilon_2 \sin\phi_2$. A good feed is completely specified by four independent parameters.

One comment: the central 4-element submatrix is a rotation matrix that represents an error in position angle of linear polarization. This depends in part on the accuracy by which the feed is mounted mechanically to the telescope; also, in-phase mutual voltage coupling between the two probes causes an apparent rotation. The off-diagonal elements $\Delta\epsilon\cos$ should be small for a well-designed, accurately-mounted feed, and also they are impossible to measure without calibration sources whose position angles are accurately known. Below we will take this factor equal to zero.

1.2. The amplifiers

Each of the two polarization channels goes through an amplifier chain with gain $G = (1 + g)$ and phase delay ψ. To characterize these we rely on careful calibration to make g and ψ small, Retaining only the first-order terms, the Jones matrix is

$$\begin{bmatrix} E_{X,out} \\ E_{Y,out} \end{bmatrix} = \begin{bmatrix} 1 + g_X + i\psi_X & 0 \\ 0 & 1 + g_Y + i\psi_Y \end{bmatrix} \begin{bmatrix} E_{X,in} \\ E_{Y,in} \end{bmatrix} \tag{9}$$

and the associated Mueller matrix is

$$\mathbf{M_A} = \begin{bmatrix} 1 & \Delta g & 0 & 0 \\ \Delta g & 1 & 0 & 0 \\ 0 & 0 & 1 & -\Delta\psi \\ 0 & 0 & \Delta\psi & 1 \end{bmatrix} . \tag{10}$$

where $\Delta g = g_X - g_Y$ and $\Delta\psi = \psi_X - \psi_Y$; also, we have eliminated a factor $(1 + g_X + g_Y)$ from the diagonal terms because it can be taken care of by intensity calibration and when applied to the off-diagonal terms it is second order.

1.3. The amplifier-feed combination

With astronomical measurements we cannot separate the feed from the amplifiers. Rather, we can only determine their product. In keeping with our previous approximation of retaining only first-order terms, this product is

$$\mathbf{M_{A+F}} = \mathbf{M_A} \cdot \mathbf{M_F} = \begin{bmatrix} 1 & \Delta g & \Sigma\epsilon\cos & \Sigma\epsilon\sin \\ \Delta g & 1 & \Delta\epsilon\cos & \Delta\epsilon\sin \\ \Sigma\epsilon\cos & -\Delta\epsilon\cos & 1 & -\Delta\psi \\ \Sigma\epsilon\sin & -\Delta\epsilon\sin & \Delta\psi & 1 \end{bmatrix} . \tag{11}$$

It is convenient and helpful that, in this approximation, the matrix elements are caused either by the feed or the amplifiers, but not both. The combination is completely specified by six independent parameters.

1.4. The azimuth-conversion Mueller matrix

This matrix relates the Stokes parameters produced by the feed to an astronomically convenient set. It is convenient to make the azimuth arm as the direction reference for our Stokes parameters, because when $az = 0°$ its position angle on the sky is also zero.

Consider Stokes Q and U. The X and Y feed probes are oriented at 45° with respect to the azimuth arm, and they provide what we call Q in the previous discussion. This is opposite to the normal definition with respect to the azimuth arm. We will fix this in the azimuth-conversion Mueller matrix $\mathbf{M_{AZ}}$. This involves not only interchanging Q and U, but also changing the sign of one of them—we are really rotating our coordinate system so we are applying a rotation matrix. We have

$$\mathbf{M_{AZ}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} , \tag{12}$$

Note that $\mathbf{M_{AZ}}$ is equal to its inverse.

1.5. The backend's Mueller matrix

We used the correlator for the backend. We generate Stokes Q from the difference and (U, V) from the product of the two polarizations. By observing astronomical sources, we found that the

Stokes Q was reversed in sign with respect to U. These signs must be consistent; otherwise the position angle behavior will go in the wrong direction (or, equivalently, U and Q won't add in phase). We could have fixed this by reversing the sign of either Q or U, but choose to reverse Q so as to maintain the original relationship between U and V.

This makes the backend's Mueller matrix $\mathbf{M_B}$ a diagonal matrix except that the element $m_{QQ} = -1$. Note that $\mathbf{M_B}$ is equal to its inverse.

1.6. The complete Mueller matrix

To completely correct the observed Stokes parameters, one first performs the primary calibration. Then one derives $\mathbf{M_B}$, $\mathbf{M_{A+F}}$, and $\mathbf{M_{AZ}}$. Applying these needs a bit of care: we need to decide whether we have derived the Mueller matrix or its inverse and, also, to apply the matrices in the proper order.

In our work, the original data were the Stokes parameters that came straight from the correlator, $\mathbf{S_{corr}}$. We applied the primary calibration to this, obtaining the observed Stokes parameters $\mathbf{S_{obs}}$.

Beginning with $\mathbf{S_{obs}}$, we applied the backend matrix to obtain the Stokes parameters that came from the amplifier-feed combination $\mathbf{S_{A+F}}$:

$$\mathbf{S_{A+F}} = \mathbf{M_B} \cdot \mathbf{S_{obs}} \ . \tag{13}$$

Note that here we are going backwards: the Mueller matrix is defined as the transfer function, i.e. the matrix that is applied to the *input* to get the *output*, but this goes the wrong way.

We then calculated $\mathbf{M_{A+F}}$ as the matrix that converts the true input to the feed $\mathbf{S_{F,input}}$ to the output $\mathbf{S_F}$ in the equation

$$\mathbf{S_{A+F}} = \mathbf{M_{A+F}} \cdot \mathbf{S_{F,input}} \ . \tag{14}$$

Combining these, we find

$$\mathbf{S_{F,input}} = \mathbf{M_{A+F}^{-1}} \cdot \mathbf{M_B} \cdot \mathbf{S_{obs}} \ . \tag{15}$$

$\mathbf{S_{F,input}}$ is a true set of astronomical Stokes parameters. However, it not defined in the conventional way. To obtain the astronomically-defined set $\mathbf{S_{astr}}$ we must apply $\mathbf{M_{AZ}}$:

$$\mathbf{S_{astr}} = \mathbf{M_{AZ}} \cdot \mathbf{S_{F,input}} = \mathbf{M_{AZ}} \cdot \mathbf{M_{A+F}^{-1}} \cdot \mathbf{M_B} \cdot \mathbf{S_{obs}} \ . \tag{16}$$

Thus the inverse of full system Mueller matrix is

$$\mathbf{M_{tot}^{-1}} = \mathbf{M_{AZ}} \cdot \mathbf{M_{A+F}^{-1}} \cdot \mathbf{M_B} \ , \tag{17}$$

and

$$\mathbf{S_{astr}} = \mathbf{M_{tot}^{-1}} \cdot \mathbf{S_{obs}} \ . \tag{18}$$

We beg the reader to forgive our unnecessary duplication of equations. It is essential to get the order of matrix multiplications correct, and our purpose is to make this absolutely clear by erring on the side of explicitness and completeness.

1.7. Certification

There are two essential checks on one's work.

First is a check on $\mathbf{M_{A+F}}$. One derives it by doing least square fits on $\mathbf{S_{obs}}$ as described below. After deriving $\mathbf{M_{A+F}}$, apply its inverse as in equation 15 and use the resulting $\mathbf{S_{F,input}}$ in the same least square fitting procedures to rederive $\mathbf{M_{A+F}}$; this new, rederived matrix should be diagonal within the errors.

Second, after applying the full correction of equation 18, verify that $\mathbf{S_{astr}}$ is consistent with **(1)** known linearly-polarized astronomical calibrators and **(2)** the radiated circular-polarized calibrator.

2. PRACTICAL CONSIDERATIONS

2.1. The primary calibration

The LBW feed is a dual-linear feed and we denote the two linearly polarized probes by X and Y. We generate Stokes Q from $X - Y$, U from $\mathrm{Re}(XY)$, and V from $\mathrm{Im}(XY)$.

First consider Q. If the relative intensity calibration of X and Y is inaccurate, then an unpolarized source appears to have different intensities and a nonzero Q' (the prime denotes the measured value). In other words, in equation 11 $\Delta g \neq 0$. There are two ways to get this relative calibration right: **(1)** Observe an unpolarized source and adjust the two cal temperatures to give $Q' = 0$; **(2)** Observe any source over a range of position angles, do a least squares fit to the equation 20 below, and adjust the cal temperatures to give $W_{Q'} = 0$. The former is quicker, but the only nonpolarized source we know of is Ori A. The latter is more reliable and, in any case, easy.

For the LBW feed, we found that adopting 10.33 and 9.62 Kelvins for the cal temperatures of the X and Y channels gave very good results. This is a total relative adjustment of 8% to the observatory-listed values of 10.76 and 9.25 Kelvins.

Next consider U and V. They are both derived from XY; the only uncertainty is the relative phase angle ϕ_{XY}. Astronomical sources have very small Stokes V, and averaging over a few gives a reliable zero. To perform this calibration, one needs a correlated noise source as a calibrator. First determine the phase angle of the cal deflection $\phi_{XY,CAL}$. Observe linearly polarized source(s) and derive the average phase $\phi_{XY,SKY}$. The difference $(\phi_{XY,SKY} - \phi_{XY,CAL})$ depends only on the mechanical details of the feed and the cable length difference between the cal as injected into X and Y channels, so it should be stable. $\phi_{XY,CAL}$ depends on the phase difference along the entire path from feed to correlator and this might change with temperature or time. Clearly, using the correlated cal allows one to continually calibrate ϕ_{XY}. After perfect calibration of ϕ_{XY}, $\Delta\psi = 0$ in equation 11.

In practice, the phase difference $(\phi_{XY,SKY} - \phi_{XY,CAL})$ is easy to estimate by observing a single well-chosen linearly polarized source over a range of parallactic angle and least-square fitting V' versus U'; with a single source the Stokes I remains constant, so in this fit the slope is the tangent of the phase angle. We refined this initial estimate by using all observed sources and solving for the system Mueller matrix as described below. We found $(\phi_{XY,SKY} - \phi_{XY,CAL}) = 2.37°$.

This angle, 2.37°, represents the cable length difference between the cal as injected into X and Y channels and it corresponds to only about 1 mm. This is very small—whoever assembled this device was very careful! In principle it doesn't matter at all whether this phase angle is small, because it can be fully corrected for. But in practice it is very convenient for it to be nearly correct. We very much appreciated this during our work.

Below, we will assume that Δg and $\Delta\psi$ are small, which in practical terms means that these two calibrations have been done accurately. (They need not be done perfectly). This is what we call the *primary calibration*. Without an accurate primary calibration you cannot proceed.

2.2. Determining the elements of $\mathbf{M_{A+F}}$

In the Mueller matrix of equation 11, the elements in the top row respresent coupling of polarized power into Stokes I. These are not easily measurable at Arecibo. As we track a source, not only does the parallactic angle change but also the zenith angle changes. The telescope gain also depends on zenith angle, particularly at high zenith angles, and this is a much bigger effect than the polarization coupling. The resulting covariance between parallactic angle and telescope gain makes the top-row elements underivable directly from observations.

The remaining matrix elements can be determined observationally, and because of the symmetry of the Mueller matrix we can then completely determine all matrix elements.

Astronomical continuum sources are linearly polarized but not circularly polarized. Using the rotation of parallactic angle as a linearly-polarized source moves across the sky makes it possible to directly measure all of the remaining matrix elements by performing least square fits as described below.

One additional remark. Arecibo's zenith-angle gain uncertainty affects not only Stokes I but the other Stokes parameters as well—they all scale directly with the telescope gain. These gain variations would affect the least square fits—not by much, but we can easily avoid this problem by using the Stokes parameters expressed in units of I instead of temperature units.

To accomplish all this we observed several sources using the "chcross" observing procedure, which samples 7 points in each of two orthogonal directions on the sky. We repetitively measured these crosses over the full tracking range of each source. We first fit a Gaussian to Stokes I. Denote this Gaussian by R_I; its height is the source deflection in Stokes I. For the other Stokes parameters we then fit the 7 points with a function of the form[2] (e.g., for Q'—there are corresponding equations for U' and V')

$$Q' = A_{Q'} + B_{Q'} R_I \ . \tag{19}$$

We emphasize that the sources are strong, so even though R_I is an observed quantity (consisting of 7 points) it is derived with high precision. However, its intensity scale is inaccurate because of the zenith angle gain variation. Because we fit $B_{Q'}$ to the observed Stokes I' Gaussian, its units are *fractional* polarization instead of Kelvins; this automatically takes care of the zenith angle gain variation for Stokes Q, and introduces no additional uncertainties because the Stokes I' values are determined with essentially zero noise.

For the full tracking range of each source, the set of coefficients $B_{Q'}$, $B_{U'}$, and $B_{V'}$ change with parallactic angle PA as (e.g. for $B_{Q'}$)

$$B_{Q'} = W_{Q'} + X_{Q'} \cos(2PA) + Y_{Q'} \sin(2PA) \ . \tag{20}$$

Here $W_{Q'}$ reflects the portion that is independent of PA and is, therefore, m_{QI}; in words, it is the fraction of Stokes I that produces a measured Q'. The coefficients X and Y are nonzero because the source is polarized, and specifically they provide the Stokes Q and U of the source as follows:

$$Q_{src} = X_{Q'} = -Y_{U'} \tag{21a}$$

[2]This equation is perfectly adequate for the present discussion and there is no reason to use anything more complicated. However, in fact we did fit a more complicated function that included spatial derivatives of the Gaussian, allowing us to derive beam squint and squash; we present this simpler form for clarity. We discuss the complete details of our actual fit in the accompanying memo (AOTM 99-02).

$$U_{src} = Y_{Q'} = X_{U'} \tag{21b}$$

These equations assume $\Delta\epsilon \cos = 0$ in equation 11.

We average the results to obtain *true* values for Q_{src} and U_{src}. This, in turn, means that for each observation we know the *true* (unprimed) Stokes parameters that are incident on the feed. Because we use fractional polarizations and assume no circular polarization, we have

$$I_{true} = 1.0 \tag{22a}$$

$$Q_{true} = Q_{src}\cos(2PA) + U_{src}\sin(2PA) \tag{22b}$$

$$U_{true} = -Q_{src}\sin(2PA) + U_{src}\cos(2PA) \tag{22c}$$

$$V_{true} = 0\ . \tag{22d}$$

These quantities are known for each observation of all sources. This allows us to directly evaluate the matrix elements using the averaging process inherent in least square fits. Also, we force each diagonal coefficient to equal unity, as it must for the case of a good feed. To be specific, we have three separate least square fits (of which the first two are really just simple averages), one for each measured Stokes parameters:

$$Q'_j - Q_{true,j} = m_{QI} \tag{23a}$$

$$U'_j - U_{true,j} = m_{UI} \tag{23b}$$

$$V'_j = m_{VI} + m_{VQ}Q_{true,j} + m_{VU}U_{true,j} \tag{23c}$$

Here the index j denotes each measurement, of which we have about 50 on each of 8 sources.

3. THE FEED'S MUELLER MATRIX

We sampled a number of highly-polarized sources, but only the following sources are sufficiently polarized and provide enough coverage in parallactic angle to be useful: CTA102, 3C454.3, 3C41, 3C399.1, 3C286, 3C274.1, P0736+01, 3C336. In future measurements of the Mueller matrix, one need not include so many sources; a small subset or even a single source is adequate. We have nearly 400 measurements and this decidedly an overkill situation. The best sources in this list are the following: 3C274.1, 3C454.3, 3C399.1, 3C286. Of these, 3C274.1 is really excellent because, with its declination of 21°, it provides a wide range of position angle coverage;

ironically, however, it is somewhat extended so it is probably unsuitable for higher-frequency measurements and, in addition, unsuitable for beam pattern measurements.

There are six independent elements for the Meuller matrix $\mathbf{M_{A+F}}$. We obtain:

$$m_{QI} = -0.0010 \pm 0.0004 \tag{24a}$$

$$m_{UI} = 0.0112 \pm 0.0002 \tag{24b}$$

$$m_{VI} = -.0055 \pm 0.0002 \tag{24c}$$

$$m_{VQ} = -0.0088 \pm 0.0024 \tag{24d}$$

$$m_{VU} = 0 \pm 0.0021 \; . \tag{24e}$$

The sixth element $m_{QU} = \Delta\epsilon \cos = 0$ by assumption.

We emphasize that these values are relevant to the system as calibrated by the 0° correlated cal *and* having been corrected for the phase difference between sky and cal. The element m_{VU} represents the error in this calibration; we iterated the primary calibration to force it to equal zero, but this was not necessary.

Stokes Q is derived from the difference between the two linear polarizations. The usual opinion is that it is therefore inaccurate and noisy. However, m_{QI} is small and its uncertainty is only twice that of m_{UI}, in which Stokes U is derived from the product. This bespeaks the accuracy of our Mueller matrix correction and, in addition, the stability of the calibration noise diodes.

4. FREQUENCY DEPENDENCE OF THE FEED'S MUELLER MATRIX

4.1. The "good feed" matrix elements in equation 15

It became rapidly apparent—just a couple of minutes into our month-long observing session—that the LBW system exhibits significant frequency dependence in the Mueller matrix elements. The 12.5 MHz-wide spectra of the four Stokes parameters all exhibited huge ~ 1 MHz-wide bumps near 1424 MHz. The properties of these bumps seemed to vary with time. Later we realized that this is well-described as a variation with source intensity, polarization, and parallactic angle. There is no easily-detectable time variability in the Stokes I bump.

Above we described the procedure by which we evaluated the five independent elements of the Mueller matrix. Before implementing this we had to choose a frequency range—the huge bump dictated that we stay away from 1424 MHz, and the 21-cm line dictated we stay away from 1420.4 MHz. In the end, our frequency range was nonsymmetrical, centered near 1418.46 MHz and its

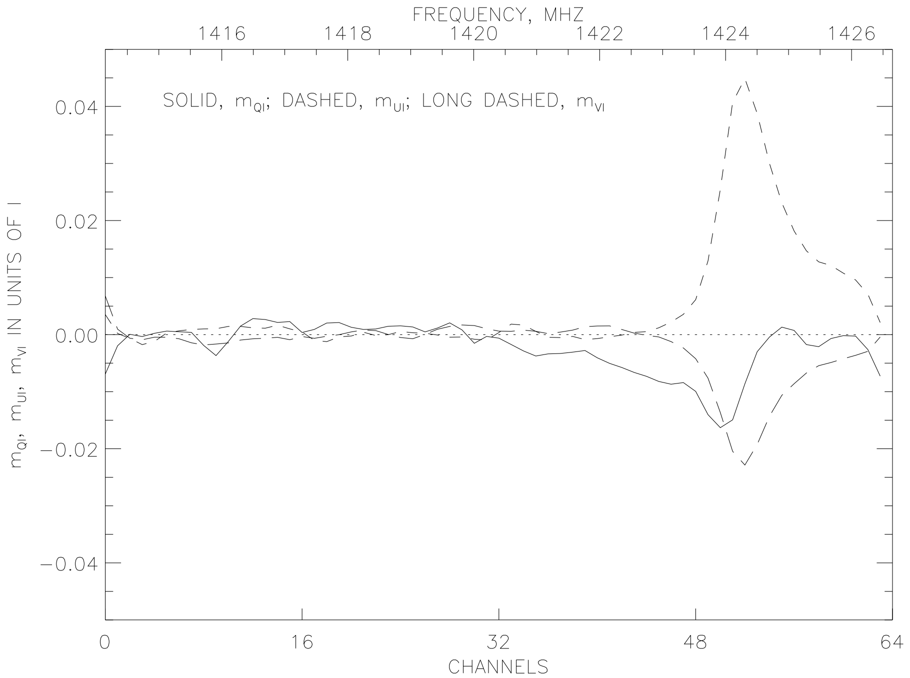

Fig. 1.— Mueller matrix elements m_{QI}, m_{UI}, m_{VI} versus channel number and frequency. Intensity units are fraction of Stokes I. The original 2048 channels have been collapsed to 64.

total bandwidth was 5.83 MHz. We designate the elements obtained for this frequency range the "continuum" elements.

We apply the Mueller continuum matrix to all observed data, including the spectra. This converts our observed (primed) Stokes parameters to true Stokes parameters, unless the matrix elements depend on frequency. We then use the above procedure to rederive the five independent matrix elements for each spectral channel. If there is no spectral dependence, then we will get zero (plus noise) for each element. Systematic departures represent spectral dependence of the feed.

There one problem with this approach. Our first-order expansion assumes that the feed is good. This is why the diagonal matrix elements are all unity and the central off-diagonal elements are zero. This assumption is decidedly not the case within the 1424 MHz huge bump, and there is no reason why these matrix elements should retain their ideal one or zero values. We will not generalize our matrix treatment because this would involve fundamental changes in our data analysis procedures. We will use our inapplicable assumption of a good feed to derive the frequency dependence of the five matrix elements in equation 15, but because the assumption is invalid the other matrix elements may also change and our results are only illustrative.

Figure 1 exhibits the lower three matrix elements in the first column, which express the degree to which Stokes I produces false polarization. The 1424 MHz bump produces false linear polarization at the level $\sim 5\%$ and false circular at $\sim 2\%$. This may not seem like much, but in fact it is very important for most continuum sources because it exceeds their levels of linear polarization. It is a rare source indeed that is as much as 10% polarized; for such a source, the apparent position angle of its linearly polarized radiation can change by tens of degrees in this bump.

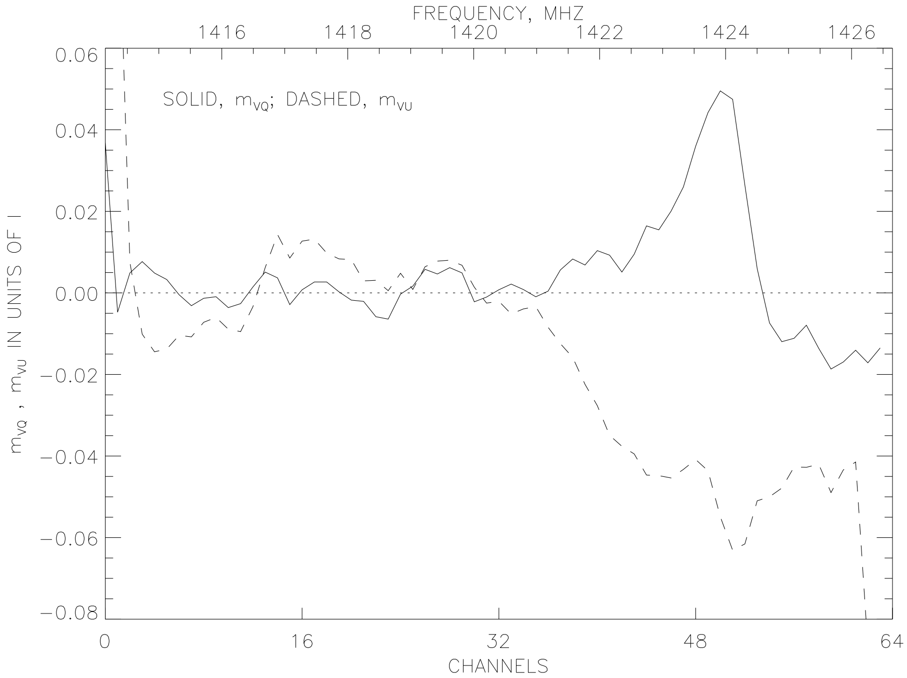

Fig. 2.— Mueller matrix elements m_{VQ} and m_{VU} versus channel number and frequency. Intensity units are fraction of Stokes I.

Figure 2 exhibits the center two matrix elements in the last row; these express the degree to which linear polarizations are converted into circular polarization. In the bump these amount to $\sim 5\%$. The element m_{VU} represents a phase difference between the X and Y polarization channels, and this reaches -0.06 in the bump, equivalent to $3.4°$.

4.2. The feed is NOT "good"!

The 1424 MHz bump affects not only the off-diagonal, "good feed" matrix elements, but also the *diagonal* matrix elements. Here we consider only the overall Stokes I gain, which is the element m_{II} in equation 3.

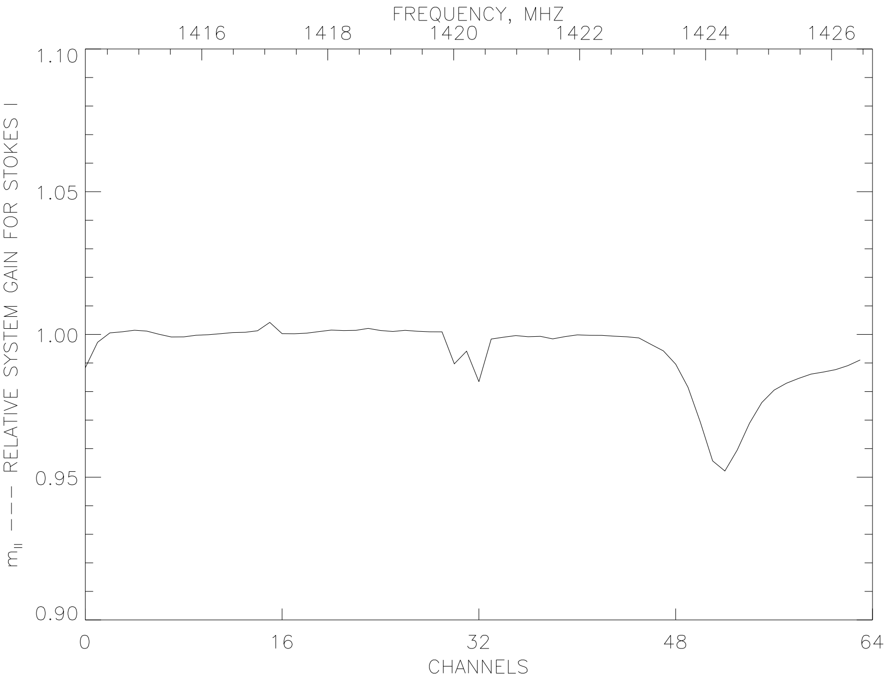

Fig. 3.— The relative system gain in Stokes I versus channel number and frequency; this is the same as m_{II} in equation 3. At each spectral point the gain is relative to the continuum gain, which is the average over the band defined in the text. The bumps at channel 16 and 32 are artifacts, a radiated test tone and the 21-cm line; the bump near 1424 MHz is real.

Figure 3 exhibits the relative system gain in Stokes I versus channel number and frequency, averaged over all observations. Recall our definition of "continuum" in section 4.1 above: the nonsymmetrical frequency range centered near 1418.46 MHz. For each observation in each spectral channel, the gain is the (source deflection in units of the continuum source deflection) divided by (cal deflection in units of the continuum cal deflection); if there is no spectral dependence, all gains are unity. The Stokes I gain is unity except at channel 16, where we were radiating a test signal; near channel 32, the 21-cm line; and in the 1424 MHz bump. The minimum gain in the bump is about 0.953.

This decreased gain was accompanied by an increased off-source system temperature in the bump. We cannot provide a plot without re-reducing all of our data because our calibration process assumed that this bump was purely a gain effect. However, we can calculate the approximate height of the bump; it was about 3.8 Kelvins. The bump is equivalent to an absorber with loss ~ 0.05 and a blackbody temperature ~ 75 K. At the peak of the bump it adds about 3.8 Kelvins to the cold-sky system temperature in Stokes I.

The bump affects the system temperatures in the other Stokes parameters, too. Figure 4 exhibits $A_{Q'}, A_{U'}, A_{V'}$ in equation 19 versus channel number and frequency. These are the off-source, blank-sky polarized system temperatures in Kelvins. The bump is prominent. We emphasize that one doesn't need any fancy least square fits to see these effects; a single 1 minute integration exhibits them clearly with high signal/noise.

Perhaps more serious than the bump is the contribution to system temperatures *away from* the bump. Outside the bump the three polarized Stokes parameters add in quadrature to ~ 3.1 K. [3] *This adds directly to the cold-sky Stokes I system temperature.* Moreover, this is only the *polarized* contribution to system temperature; presumably there is an additional unpolarized component, meaning that *away from the bump* these feed imperfections raise the system temperature by *at least* 3.1 K.

This correlated noise might come from losses in the feed or, alternatively, from reflections—for example, the X receiver temperature radiated towards the feed and reflecting back to enter the Y receiver. Whatever the cause, the system temperature is degraded.

It is a pleasure to thank Phil Perillat for providing the excellent observing software and critical, stimulating comments; Mike Davis for general assistance and advice; and Kiriaki Xilouris for introducing me to the Mueller matrix.

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[3]The cold-sky Stokes Q is the difference in system temperature between X and Y and might be only a difference in receiver temperature.

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Table 1. POLARIZATION CALIBRATORS USEFUL AT ARECIBO

Source	α_{1950}	δ_{1950}	S, Jy	P_s, Jy	%Pol	θ_s
3C33	01 06 14.2	13 03 37	12.53	1.81	7.09 ± 0.42	68.0 ± 0.9
3C41 AO	01 23 54.8	32 57 36.0	3.5	0.42	6.0	48.9
3C98	03 56 11.0	10 17 41	10.25	1.05	5.10 ± 0.15	71.5 ± 0.8
3C123	04 33 55.2	29 34 14	45.16	0.45	0.50 ± 0.19	146.8 ± 5.5
3C138	05 18 16.5	16 35 26	8.88	1.11	6.81 ± 0.41	176.5 ± 1.1
3C144-TAU	05 31 31.0	21 59 17	895.50	14.68	0.82 ± 0.08	87.0 ± 2.5
3C227	09 45 07.8	07 39 09	7.18	0.74	5.14 ± 0.37	145.2 ± 1.0
P0736+01 AO	07 36 42.6	01 44 00.0	2.9	0.48	8.2	103.4
3C270	12 16 51.2	06 06 13	17.20	2.62	7.62 ± 0.30	125.1 ± 2.3
3C273	12 26 32.9	02 19 39	49.77	1.23	1.24 ± 0.04	149.8 ± 0.9
3C274	12 28 17.8	12 39 50	213.56	4.27	1.00 ± 0.05	144.5 ± 5.6
3C274.1 AO	12 32 57.0	21 37 06.0	2.64	0.67	12.8	158.6
3C274.1	12 32 57.0	21 37 06	2.64	0.77	14.52 ± 1.70	151.2 ± 1.7
3C286 AO	13 28 49.7	30 46 02.0	14.78	2.86	9.7	28.8
3C286	13 28 49.7	30 46 02	14.78	2.74	9.52 ± 0.16	30.8 ± 0.1
P1414+11	14 14 27.3	11 02 16	4.14	0.82	9.89 ± 0.22	25.5 ± 3.1
3C336 AO	16 22 33.5	23 52 06.0	2.7	0.15	2.7	29.1
3C348	16 48 40.1	05 04 28	43.69	1.37	1.57 ± 0.17	63.1 ± 1.5
3C399.1 AO	19 14 00.0	30 14 23.0	2.7	0.56	10.3	53.6
3C433 sp	21 21 30.6	24 51 18	11.68	1.67	$7.1\pm$ ···	$136.0\pm$ ···
CTA102 sp	22 30 07.7	11 28 23	6.37	0.65	$5.1\pm$ ···	$105.3\pm$ ···
3C454.3 AO	22 51 29.4	15 52 56.0	13.56	1.57	5.8	70.6
3C454.3	22 51 29.4	15 52 56	13.56	2.09	7.69 ± 0.55	67.9 ± 1.0

Note. — Column 1 is the source name, column 2 and 3 the 1950 equatorial coordinates, column 4 the flux density S in Jy ($S = \frac{\text{Stokes } I}{2}$), column 5 the polarized flux density $P_s = (Q_s^2 + U_s^2)^{1/2}$ in Jy, column 6 the percent polarization (defined as $\frac{100 P_s}{2S}$), column 7 the position angle. Positions and fluxes are from Heiles and Fisher (1999) or BDFL(1972).

Note. — "AO" after a source name designates an Arecibo observation; otherwise data are from the NRAO 140 foot (Heiles and Fisher 1999). Arecibo data are generally more accurate.

Note. — Green Bank results are straight averages from Heiles and Fisher's (1999) table; errors are not very meaningful, and are in most cases underestimates. For the Green Bank data, the letters "sp" means that the source was observed during the Spring 1998 observing period; uncertainties are not available and the data somewhat less accurate than in Jan 1999.

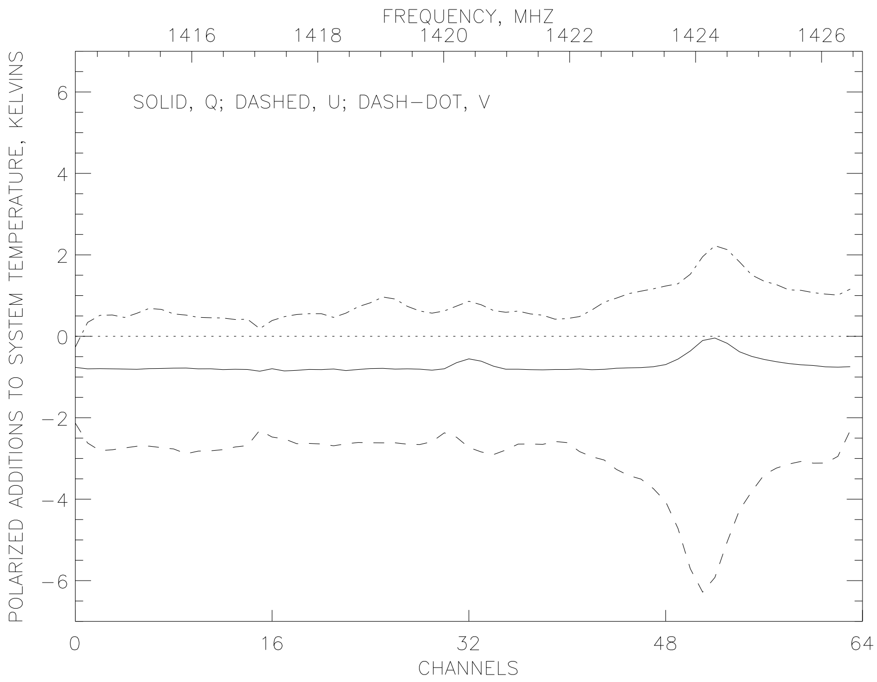

Fig. 4.— $A_{Q'}, A_{U'}, A_{V'}$ in equation 19 versus channel number and frequency. These are the off-source, blank-sky polarized system temperatures in Kelvins.