RADIO INTERFEROMETRY AT 12.2 GHZ
January 13, 2006

1. SYNOPSIS

This course starts with a bang and keeps up until the last day of class. *Don't procrastinate!!* This lab will run for five weeks. During each week you, as an individual, will need to produce written solutions to problems that are relevant to the lab measurements; and you, as a group, will need to make measurements. For this lab, the five weeks will go something like this:

1. Groups will observe the Sun and Moon with the interferometer. Individuals will use rotation matrices to calculate when various objects of interest are up and hand printed versions of the software and the results.

2. Groups will observe horizon-to-horizon tracks of weak point sources. Individuals will calculate Fourier power spectra of the previous week's data and also this week's data and hand in printed versions of the plotted results.

3. Groups will observe horizon-to-horizon tracks of the Sun. Individuals will write least-squares fitting software to get accurate source positions. and hand in printed veresions of the software and derived positions for the weak point sources.

4. Groups will observe horizon-to-horizon tracks of the Moon and finish any observations they haven't completed. Individuals will derive brightness distributions for the Sun and hand in the results.

5. Individuals will derive brightness distributions for the Moon and use LATEX to write full-fledged lab reports that cover the whole enchilada.

IMPORTANT!! Note that each week you are doing something completely new, and that you have software to write and results to derive to hand in.

2. OUR INTERFEROMETER

With our interferometer, which operates at 12.2 GHz, our attention focuses on small sources. Our interferometer has a relatively short baseline and the fringe spacing is larger than the size of all sources (except the Sun and Moon). So for our interferometer all of these sources look like "point sources", for which all the radiation appears to come from a single point—just like a star in optical astronomy. With the longer baselines of research-class interferometers/arrays comes increased angular resolution, and all of these sources become finite in angular size, which means that the telescope arrays can map the sources using Fourier techniques. Some types of source are so small that mapping them requires interferometers with baseline lengths comparable to the Earth's diameter, a technique called "Very Long Baseline Interferometry" (VLBI). And some sources, such as pulsars, are even too small for VLBI!

We have a *multiplying* interferometer: the signals from the two telescopes are multiplied together. This is equivalent to adding the signals and then squaring. Actually, it's much better because the average value of the product is zero, so any detected signal has to come from the sky instead of the instrument. So when we look at a source, the fringe amplitude has no zero offset and the amplitude is directly proportional to the source flux.

Interferometers provide not only the fringe amplitude but also the fringe phase and frequency (see *Commentary* below). If you know the baseline, these are direct indicators of the source declination. With our 10-m baseline we can get a fringe spacing $\sim 10'$, and with a horizon-to-horizon measurement we can measure the declination 100 times more accurately. In addition, we can achieve partial angular resolution on the Sun and Moon and measure their diameters and limb darkening.

3. CONTINUUM SOURCES

For the first part of the lab we will concentrate on measuring source positions. Our telescopes are small, so there are only a few sources that are powerful enough for us. The strongest continuum sources are

name	r.a.(1950)	dec(1950)	r.a.(1998)	dec(1998)	S_{Jy}
3C144 (Crab Nebula)	$05^h31^m30^s$	$21°58'$	$05^h34^m23^s$	$22°00'$	~ 496
Orion Nebula	$05^h32^m44^s$	$-05°25'$	$05^h35^m05^s$	$-05°23'$	~ 340
3C274 (Virgo A)	$12^h28^m18^s$	$12°40'$	$12^h30^m44^s$	$12°24'$	~ 34
M17	$18^h17^m33^s$	$-16°12'$	$18^h20^m19^s$	$-16°11'$	~ 500
3C405 (Cygnus A)	$19^h57^m45^s$	$40°36'$	$19^h59^m24^s$	$40°44'$	~ 120
3C461 (Cas A)	$23^h21^m07^s$	$58°34'$	$23^h23^m17^s$	$58°50'$	~ 320
SUN	varies	varies			
MOON	varies	varies			

You'll need to precess these coordinates to the current equinox; use the GSFC IDL procedure *precess*. We've supplied the 1998 positions as a guide.

3.0.1. The Point Sources

Our goal is to measure their absolute declinations as accurately as possible. The positions quoted above are accurate to only about 1 arcminute. Declinations can be measured on an absolute basis with interferometry by least-squares fitting the fringe phase to the hour angle; see *Commentary* below. With our fringe spacing of $\sim 10'$ we might be able to measure declinations to an accuracy $\sim 20''$—maybe better.

Also we want to measure the difference in right ascension between the sources. We can't measure the *absolute* right ascensions because the right ascension coordinate has no naturally-determined zero point; rather, its zero point is defined arbitrarily by convention (and it changes with time, too). In contrast, declination has a naturally-determined zero point: the equator. But we can measure the *relative* right ascension of one source with respect to another with high accuracy. At least we can do this in principle; in practice it's harder than measuring only the declinations.

We'll want *each group* to pick a source and obtain a horizon-to-horizon observation of the fringes. Groups should compare results to see if you can get the differences in right ascensions (we've never successfully done this before!). And we'll want *each person* to measure the source's declination by least-squares fitting the horizon-to-horizon track of fringe phase and amplitude. Compare your results with other members of your group. Help each other out, but *each person should write her/his own software.*

There are some considerations in picking sources, and we will save you some frustration by telling you about them beforehand:

1. The fringe frequency depends on $\cos\delta$. This means that you don't measure δ directly, but rather $\cos\delta$. Can you get accurate declinations for sources near $\delta = 0$?

2. Southern sources are difficult for horizon-to-horizon tracks because of the presence of the Campanile.

3. Our telescope pointing for Northern sources may be inaccurate, leading to loss of signal/noise.

4. Our observing frequency is in the TV satellite band. Geostationary TV satellites sit in the southerly skies and generate strong signals that can enter the dish sidelobes and produce fringes.

3.0.2. The Sun and Moon

The Sun and Moon are special cases, for two reasons. First, their positions change from day to day—and for the Moon, the change in just an hour is significant[1]. Second, they are both extended sources, not point sources. This causes their fringe amplitudes and phases to change with time, in a manner that depends on their brightness distribution[2]; this makes the determination of accurate positions a bit tricky, but we'll ignore that detail for now.

You can get the Sun and Moon positions from our IDL procedures *isun* and *imoon.* For the Moon, pay particular attention to the *geocentric* option, which removes the parallax correction. The parallax effect is far greater for the Moon than the Sun—so large that unless you correct for it, the Moon will probably lie outside the telescope main beam! A previous lab TA Erik Shirokoff worked very hard to get the moon position, including parallax, really right; he wrote *imoon.* We think it works. We *hope* it works!

Given these difficulties, why bother with the Sun and Moon? Because they are *bright.* We include them because they provide huge signals, which is ideal for testing your observing setup and your reduction software. In particular, the Sun is so bright that you'll get a huge signal/noise and you should be able to estimate an accurate declination in the first few minutes. And if you don't see the Sun, you *know* you're doing something wrong!

3.0.3. Some Astrophysics

Here's a bit of physical information. M17 ("M" for Messier) and Orion are HII regions—places where hot stars have produced warm ($T \sim 10^4$ K) ionized gas, where the electrons flying past the protons produce free-free (bremsstrahlung) radiation. The other sources radiate in synchrotron radiation—relativistic electrons in a magnetic field. For these sources, the source designation 3CXYZ designate source number XYZ from the third Cambridge (England) catalog; in the early 1960's, Cambridge radioastronomers produced the first reliable comprehensive catalog of strong radio sources in the Northern hemisphere. The Crab Nebula (also called Taurus A) is powered by the Crab pulsar, and is a $\sim$ 1000 yr-old supernova remnant in the Galaxy about 1 kpc distant; Cas A is another supernova remnant, *not* powered by a pulsar, $\sim$ 300 yr old and about 2.5 kpc distant; both of these are expanding rapidly, as befits their young ages, and Cas A (in which a pulsar does not constantly replenish the electrons) is gradually getting dimmer. Virgo A is a "peculiar" elliptical galaxy about 11 Mpc distant, while Cygnus A is a giant elliptical galaxy 220 Mpc distant. Cyg A is a powerful radio source—it's 10^5 times further than Cas A and just as bright! The study

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[1] Use what you already know to make an order-of-magnitude estimate of how far the Sun and Moon move in one day!

[2] These changes in fringe properties are exactly what's necessary to map the sources!

of the mechanism by which this enormous power is generated, which implies enormous energies, has led to the current awareness of and interest in *high-energy astrophysics*.

The Sun also emits free-free radiation, just like the HII regions. But the Moon is another story. Contrary to what you might expect, at radio wavelengths it *doesn't* shine by reflected sunlight. Rather, its emission is blackbody radiation from its solid surface. Its surface is heated by sunlight, and at short wavelengths (but not at long ones) there's a big difference between the temperature of the sunlit and dark parts of the Lunar surface. You can tell a lot about its surface properties from the polarization of the radiation and also from its time variability as the surface heats up from sunlight and cools off from darkness—just like the Sahara.

4. MEASURING ACCURATE DECLINATIONS

4.1. General Description

We measure declinations from the fringe frequency, which depends on the baseline orientation, baseline length, declination, and hour angle (all these go into the *projected baseline*. If we observe a source horizon-to-horizon, the projected baseline changes a lot, and so does the fringe frequency. We take those data and do a least-squares fit to derive the most accurate declination from our data.

FIRST WEEK: Before doing the weak sources in the Table, do the Sun for a much shorter time, say an hour. This will give you a huge signal that you can look at visually, think about, and derive the approximate declination with pencil and paper. Then later you can write software to do the same, and make sure you get the right answer. Next, during the second week, do the horizon-to-horizon track of a source from the Table.

Below we'll distill the formulae given in the appendix of the recommended reference to the nice, straightforward case of an east-west baseline of length B, for which the only nonzero baseline component is B_y. Our interferometer has an east-west baseline, so our distilled formulae will be applicable to your measurements.

4.2. The Details

The two interferometer telescopes have different distances from the source. The difference can range from zero (if the source is overhead) to nearly the the full baseline (if the source is near the horizon). This distance difference is tiny compared to the distance to the source, but it's important!

It's convenient to think of the different distances in terms of relative path delay in *time* units for the two telescopes; we call this the *geometrical* path delay τ_g. But don't forget! The signals travel through a lot of electronics before they get multiplied and the two paths aren't of equal length, so there is an additional relative delay from the difference in cable length τ_c. The total relative delay is the sum of the two,

$$\tau_{tot} = \tau_g(t) + \tau_c \ . \tag{1}$$

We explicitly include the fact that τ_g is a function of time—it depends on the hour angle. In contrast, τ_c is independent of time (unless somebody changes the cable setup. . .).

We don't know τ_c (but the least-squares process can tell us what it is). However, we do know τ_g because it's just geometry—the geometry discussed in the reading. For the east-west baseline, we have

$$\tau_g(t) = \left[\frac{B_y}{c}\cos\delta\right]\sin h \ . \tag{2}$$

The output of the interferometer is the product of what the two telescopes see. If they are looking at a monochromatic source then the voltages for the two telescopes are

$$E_1(t) = \cos(2\pi\nu t) \tag{3}$$

$$E_2(t) = \cos(2\pi\nu[t + \tau_{tot}]) \ . \tag{4}$$

and the product is the interferometer fringe output

$$F(t) = \cos(2\pi\nu t)\cos(2\pi\nu[t + \tau_{tot}]) \ . \tag{5}$$

There's a trigonometric identity that allows us to write this in terms of the sum and difference of the two arguments[3]. The sum term varies rapidly with time and averages to zero; it's the difference term we want, so if we exclude the sum term (and forget about the factor $\frac{1}{2}$) we get

$$F(t) = \cos(2\pi\nu[\tau_g(t) + \tau_c]) \ . \tag{6}$$

We want to use a least-squares fit to find the values of quantities that comprise the argument of the cosine. If you have done least-squares fitting before (and if you remember anything about it!) you'll realize that the straightforward least-squares fitting technique won't work on this type of problem. We can simplify things for the fitting process by using another trig identity[4] and writing this as the sum of two trig functions

[3] $\cos(A)\cos(B) = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

[4] $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$F(t) = \cos(2\pi\nu\tau_c)\cos\left[2\pi\nu\left(\frac{B_y}{c}\cos\delta\right)\sin h\right] - \sin(2\pi\nu\tau_c)\sin\left[2\pi\nu\left(\frac{B_y}{c}\cos\delta\right)\sin h\right] \ . \qquad (7)$$

This may not look simpler! But it is, because for the purposes of least-squares it involves only a *single* variable in the trig function arguments—the combination of variables $\left(\frac{B_y}{c}\cos\delta\right)$ (we are assuming that we know the right ascension well enough to get a good value for h).

To proceed with least-squares, replace $\cos(2\pi\nu\tau_c)$ and $\sin(2\pi\nu\tau_c)$ by two "unknown constants" A and B, respectively; assume that they are unrelated and solve for them using the standard least-squares process. Also, it's convenient and intuitive to make the substitution

$$\nu\left(\frac{B_y}{c}\cos\delta\right) = \left(\frac{B_y}{\lambda}\cos\delta\right) \qquad (8)$$

which expresses the delay in units of wavelength, and thus the "number of turns" or phase. These substitutions give

$$\boxed{F(t) = A\cos\left[2\pi\left(\frac{B_y}{\lambda}\cos\delta\right)\sin h\right] - B\sin\left[2\pi\left(\frac{B_y}{\lambda}\cos\delta\right)\sin h\right] \ .} \qquad (9)$$

Let's take a moment and reflect on this complicated-looking equation, focussing on just the first term (because the second is identical except it's a sine instead of a cosine). We want to develop the concept of a *local fringe frequency* f_f. The argument of the cosine is a constant $C = \left[2\pi\left(\frac{B_y}{\lambda}\cos\delta\right)\right]$ multiplied by $\sin(h)$. Now h is the hour angle and increases monotonically with time, so we can regard it as time[5]. Now C is multiplied not by h itself, but rather by $\sin(h)$, which is a nonlinear function of time. The product $C\sin(h)$ is the argument of the cosine term and makes it oscillate back and forth, but at a frequency that depends on time as h changes.

This leads to the concept of the local fringe frequency. To see this, expand the hour angle term $\sin(h)$ into a Taylor series centered on the current hour angle of the source h_s:

$$\sin h = \sin(h_s) + \Delta h\left.\frac{d\sin(h)}{dh}\right|_{h_s} = \sin(h_s) + \Delta h\cos(h_s) \qquad (10)$$

The local fringe frequency is contained in the second term because, for a small region around h_s in equation 9, $F(t)$ varies as $f_f\ \Delta h$, where the local fringe frequency f_f is

$$f_f = \frac{C}{2\pi}\cos(h_s) = \left(\frac{B_y}{\lambda}\cos\delta\right)\cos(h_s) \ . \qquad (11)$$

[5]Except that its units are radians. This is no problem: 24 hours is 2π radians.

This is the local fringe frequency in cycles per radian on the sky. If you want to turn it into cycles per hour coming out of the interferometer's multiplier, multiply by $\frac{2\pi}{24}$; or cycles per minute, multiply by $\frac{2\pi}{60\times24}$; etc. At the meridian ($h_s = 0$), the fringe frequency is $[f_f = 0.029\cos\delta]$ cycles per second and the period is $\frac{1}{f_f} = \frac{35}{\cos\delta}$ seconds.

Now let's return to the least-squares solution of equation 9. Make a guess at the proper value of $\left[\frac{B_y}{\lambda}\cos\delta\right]$ and, also, *adopt* a value for the right ascension α (required so that you can compute h from the local sidereal time LST). With this, you know the arguments of the trig functions and this means that you can solve for A and B using the standard least-squares process; be sure and *save the sum of the squares of the residuals.* Then change the guessed-at value of $\left[\frac{B_y}{\lambda}\cos\delta\right]$ and do it again. Do this a number of times and plot the sum-of-squares versus the guessed-at value of $\left[\frac{B_y}{\lambda}\cos\delta\right]$. The best value of δ is where the sum-of-squares is a minimum. What is this? *It's [brute-force] least-squares!* You might try doing the same kind of iteration with the right ascension, but my guess is that it won't make much difference. Then use A and B to calculate the cable length difference τ_c.

Notice!!! The parameter of interest is the declination δ. In contrast, we suggest solving for the combination $\left[\frac{B_y}{\lambda}\cos\delta\right]$. Why is this? When you do the least-squares fitting you need to know the baseline length, the distance between the telescopes. You have to measure this! And if you do it wrong, then... well, suppose your measurement of the baseline is too small—by a factor of 100 (because you measured the length in cm instead of m!). Look at the function you fit: it has the baseline multiplied by $\cos(\delta)$. *cos*(δ) can never be larger than one. So if you use a baseline that is too small in your solution, the best value for $\cos(\delta)$ might be larger than unity—meaning that *no* declination will give a satisfactory solution! The most straightforward way around this is to fit not for the declination, but for $\left[\frac{B_y}{\lambda}\cos\delta\right]$, and then after deriving this parameter divide by the baseline B_y to find the best value of $\cos(\delta)$. This allows $\cos(\delta)$ to be greater than 1 in your trial fits—which might be especially important for sources near the equator.

If you do this least-squares reduction for a bunch of sources, you will get all of their declinations. However, you will find different cable lengths for each source. Clearly, this is a problem because nobody has changed the setup during your measurements (you hope). The different cable lengths mean that the *adopted* right ascensions are not all mutually consistent. You need to change the relative right ascensions in such a way that the fits all give the same cable delay. This will require comparing results among groups and doing some iteration. This is a challenge: never before in this lab have the relative right ascensions been reliably determined. Have fun!

4.3. Least-squares fitting: Commentary

Above, we discussed the technique least-squares fitting for the declination. This is not a straightforward problem because $\frac{dF(t)}{d\delta}$ is not equal to a constant—rather, it depends on time. This means you have to do what's called a *nonlinear least-squares fit.* The general technique for this—

which is not what we recommended—is: you guess a value for the declination, expand the equation for $F(t)$ in a one-term Taylor series about this guess, and solve for the correction. If you're lucky the process converges rapidly. If you have time, you might try this!

We suggested a different technique above, one I call the "brute force technique". It simply does the least-squares fitting process by iteration and inspection: you minimize the sum-of-squares of the residuals yourself instead of by a mathematical procedure. This minimization is exactly what the least-squares fit does! If there's only a *single* variable involved, then it's straightforward to use the brute-force technique. But if there's more than one, then things get complicated rapidly. This is why we rewrote the equations above so that there was only a single variable involved in the nonlinear fit.

5. MEASURING 1-D BRIGHTNESS DISTRIBUTIONS: THE FIRST STEP OF MAKING MAPS

When we think of a time-variable signal, we think of frequency as being cycles per second—and its inverse, the period, is in seconds, the number of seconds that separates adjacent peaks of the sine wave.

The interferometer projects a giant sine wave on the sky. Its frequency, which changes with position, is measured in cycles per radian—and its inverse, the period, is the angular separation of adjacent peaks, measured in the angular units of radians. You can, of course, also think of frequency in terms of cycles per degree or cycles per arcminute, with the corresponding periods ("fringe separation") in units of degrees or arcminutes.

When we observe with a range of baseline lengths and orientations, the giant sine waves in the sky have corresponding ranges of frequencies and orientations. We sample brightness of the sky in *Fourier* space. The fringes at each baseline length and orientation have amplitudes and phases. To recover the brightness of the sky in *real, angular* space, we measure as many Fourier components as we can and take their Fourier transform. If we had complete sampling in Fourier space, we would recover the true brightness distribution. In real life, we have *in*complete sampling, so we recover a distorted representation of the true distribution. There is a whole literature of techniques for minimizing this distortion, the most prominent being "cleaning" and "maximum entropy". Full-fledged research arrays, such as the Very Large Array (VLA) in New Mexico, rely on these techniques to map the sky.

In our case we have just two dishes along an east-west line. The effective baseline length changes as the source rises higher in the sky, and as long as the source is away from declination $\delta = 0^\circ$ the orientation of the baseline also changes, at least to some degree. We will map the Sun and Moon, which never get very far from $\delta = 0^\circ$, so effectively we have only a 1-d sampling of the source with a range of baseline lengths.

Figure 1 illustrates this 1-d concept. It presents a generic circular source of uniform brightness in the sky, which we call the MUN—a bastardization of the MOON and the SUN[6]. The top panel shows the MUN in the sky as it really is: the fringes cover the 2-d object. At the bottom, we integrate along vertical strips to get the 1-d brightness distribution—the vertically-integrated 1-d equivalent, in which both the brightness distribution and the fringes depend on only one coordinate.

This one coordinate is the horizontal direction in the bottom panel of the Figure. In real life this is hour angle because we have an east-west interferometer and our source is at low declination—meaning that the baseline projected on the sky is mainly east-west, the direction of hour angle. Thus we denote this direction by the letter h.

As the sky rotates, the source moves through the fringe pattern to give the fringe response $R(t)$. The time t is the same as the hour angle h. For a point source, $R(h) = F(h)$; for an extended source, we have to integrate over the extent of the source. Let $I(h - h_s)$ be the 1-d intensity distribution in the sky. That is, on the bottom panel of Figure 1, $I(h - h_s)$ is the intensity of the source (vertical direction) and $h - h_s$ the horizontal coordinate—the hour angle h relative to the hour angle of the source center h_s. To be precise, the source center is the weighted mean of the position, i.e.

$$h_s = \frac{\int I(h) h dh}{\int I(h) dh} \ . \tag{12}$$

Now at any one time (hour angle h) the fringe pattern $F(h)$ covers the source and the interferometer response $R(h_s)$ is the integral of the fringe pattern times the source intensity distribution, so we have

$$R(h_s) = \int F(h) I(h - h_s) \ dh \tag{13}$$

Now write $U = \left(\frac{B_y}{\lambda} \cos\delta \right)$ and use equation 9 to write

$$R(h_s) = A \int I(h - h_s) \cos(2\pi U \sin h) \ dh + B \int I(h - h_s) \sin(2\pi U \sin h) \ dh \tag{14}$$

Our source occupies a small angle, so we can expand h around h_s by writing $[\Delta h = h - h_s]$. We use equation 10 to expand $\sin h$, and also we consider only the first term to eliminate clutter. The first term becomes

$$R(h_s) = \int I(\Delta h) \cos[2\pi U (\sin h_s + \Delta h \cos h_s)] \ d\Delta h \tag{15}$$

[6]In truth, it represents neither, because neither the Sun nor the Moon have uniform surface brightness.

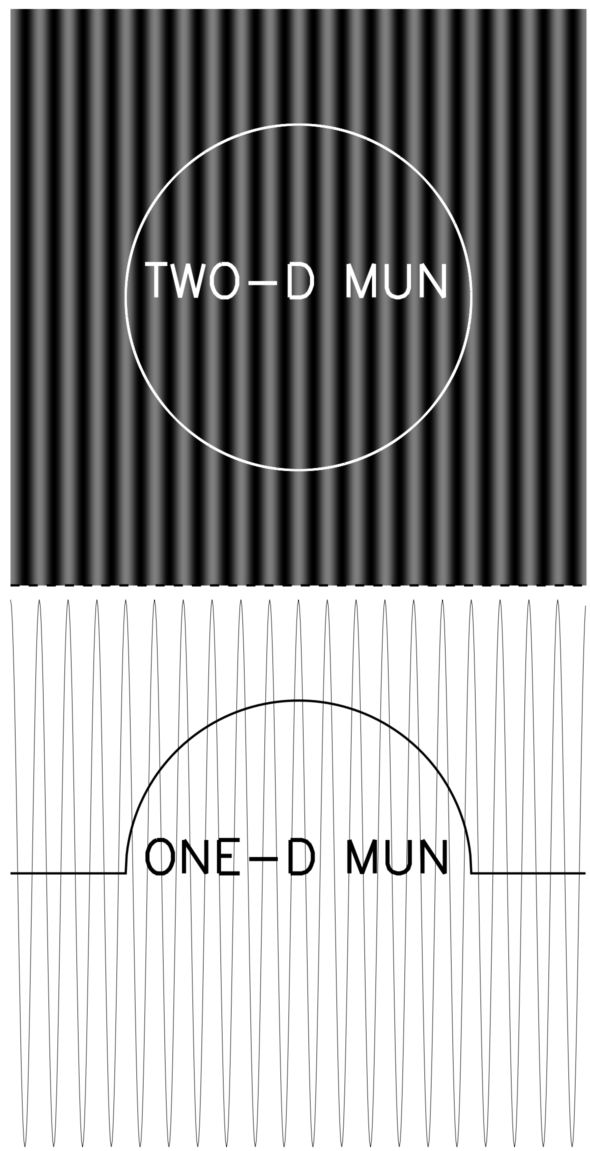

Fig. 1.— The 2-d and 1-d MUN. At the top, we see the situation in the sky as it really is: the fringes cover the 2-d object. At the bottom, we integrate along vertical strips to get the 1-d brightness distribution and, also, the fringe amplitude (which goes from –1 to +1).

Inside the integrand we have a sum of a sin and cos term. This can be expressed as a product using the trig identity $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$:

$$R(h_s) = \cos\left[2\pi U \sin(h_s)\right] \int I(\Delta h) \times \cos\left[2\pi U \cos(h_s) \Delta h\right] \; d\Delta h - \tag{16}$$

$$\sin\left[2\pi U \sin(h_s)\right] \int I(\Delta h) \times \sin\left[2\pi U \cos(h_s) \Delta h\right] \; d\Delta h$$

We imagine $I(\Delta h)$ as being composed of little slices in hour angle, with each little slice of the source characterized by its position offset Δh and its intensity $I(\Delta h)$. For our source (the MUN), we assume the that $I(\Delta h)$ is symmetric (This also retains our algebraic sanity). This means that in the above equation we need retain only the top line, which is even (cosine term in Δh). This gives the *important equation*

$$R(h_s) = \underbrace{\cos\left[2\pi U \sin(h_s)\right]}_{Fringe\ of\ a\ point\ source} \times \underbrace{\int I(\Delta h) \cos\left[2\pi U \cos(h_s) \Delta h\right] d\Delta h}_{Modulates\ (multiplies)\ the\ fringe} \tag{17a}$$

which can also be (perhaps more transparently) written

$$\boxed{R(h_s) = \underbrace{\cos\left[2\pi U \sin(h_s)\right]}_{Fringe\ of\ a\ point\ source} \times \underbrace{\int I(\Delta h) \cos\left[2\pi f_f \Delta h\right] d\Delta h}_{Modulates\ (multiplies)\ the\ fringe}} \tag{17b}$$

Note the structure of equation 17. It consists of two factors. The first is the fringe pattern of a point source located at hour angle h_s. The other multiplies (modulates) this function. The modulating function is a trig function. It can (and does!) go through zero. *The locations of these zero points provide crucial information about the source structure.* When it goes through zero, it changes the sign of the oscillatory first term; you can also think of this as a phase change by 180°.

More generally, the modulating function is the Fourier transform of the source distribution. *The modulating function contains the essential information about the angular structure of the source.* As an example, Figure 2 (top panel) shows two examples of 1-d brightness distributions, a flat and a cosine distribution. The bottom panel shows the Fourier transforms. You'll measure the fringe modulation functions of the Sun and Moon and figure out their 1-d brightness distributions!

6. REFERENCE READING

6.1. On Interferometry and Aperture Synthesis

The appropriate reference for our purposes is the article *Interferometry and Aperture Synthesis*, which is chapter 10 of the book *Galactic and Extragalactic Radio Astronomy, First Edition.* The authors are Fomalont and Wright; Melvyn Wright is a research scientist in our radio lab here at Berkeley and is a real expert. This chapter is excellent, providing the basics without excessive detail (although it has more than we need). If you want more depth than we provide here, we suggest

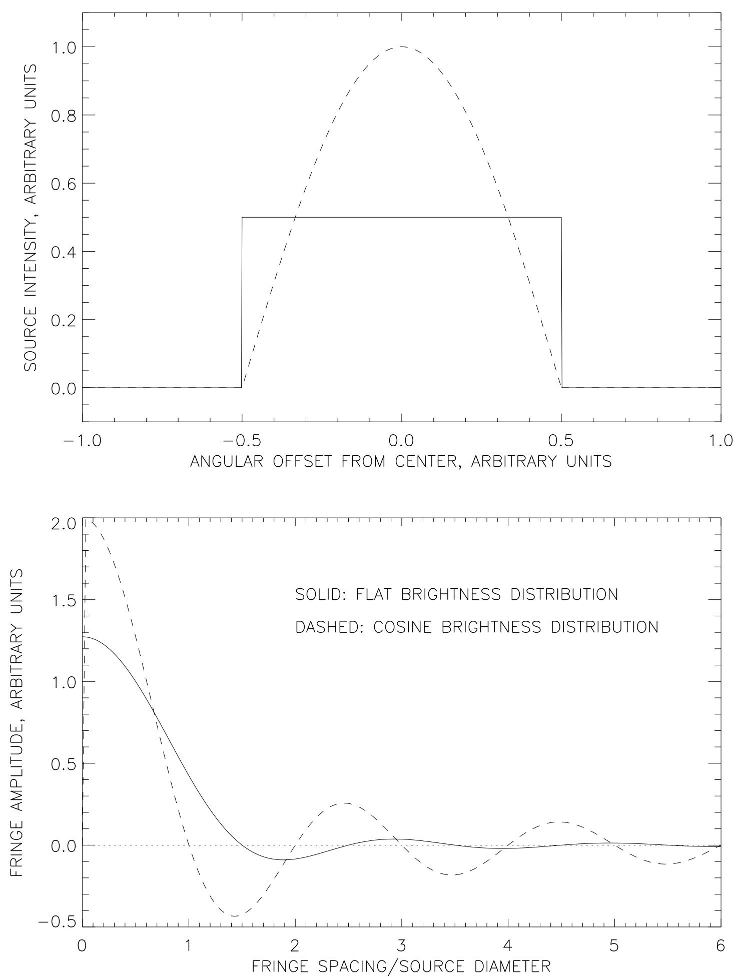

Fig. 2.— Examples of 1-d brightness distributions and their Fourier transforms. Top panel: the brightness distributions. Bottom: the Fourier transforms. *Note:* The labels for the solid and dashed lines on the bottom (but not the top) are flipped.

the following sections of this chapter: **(1)** §10.1.3, which describes a two-element interferometer; section e of this chapter is on polarization and you can skip it; **(2)** §10.2.1 and §10.2.2, which describe "a working interferometer"; and **(3)** Appendix II, which describes the geometrical details.

There is a scaling mistake in their equation for the fringe frequency ν_f: their equation needs to be multiplied by the rotation rate of the Earth in radians per second. For example, for an east-west baseline of 343.8 wavelengths looking at declination zero on the meridian, the fringe frequency on the sky is 343.8 fringes per radian. This means that the fringe spacing on the sky is $\frac{1}{343.8}$ radians or 10 arcmin; it takes the Earth 40 seconds of time to turn through 10 arcmin, so the fringe period in this case is 40 seconds and $\nu_f = .025$ Hz. More generally, for this east-west interferometer the fringe frequency is $\nu_f = .025 \cos\delta \cos h$ Hz, where δ is the declination and h the hour angle.

If you want to know really everything and in complete mathematical detail, read the book *Interferometry and Synthesis in Radio Astronomy* by Thompson, Moran, and Swenson. But such detail is more than we want and can be overwhelming. Our main interest is the geometry—how the baseline projects on the $u-v$ plane. This is in chapter 4, and the most important sections for us are chapter 4.2 and 4.4.

6.2. On the Astrophysics of Radio Synchrotron Emitters

Supernova remnants and external galaxies—radio continuum sources of the types listed above—shine by the radiation of relativistic electrons gyrating in a magnetic field, known as synchrotron radiation. In the Crab Nebula, the electrons are accelerated by the pulsar's spin and spewed out into the nebula. In Cas A, the electrons are produced behind the fast shock wave produced by the explosion. In the external galaxies, the ultimate source of the electrons involves acceleration of electrons near the black hole at the center; the electrons are then spewed out to extragalactic space in narrow collimated jets, and produce large "emission lobes" at the end of the jets. For information on both types of source and some beautiful pictures, see chapters 10 and 13 in *Galactic and Extragalactic Radio Astronomy, second edition* (1988, ed. G.L. Verschuur and K.I. Kellermann).