MIXERS for HETERODYNE SYSTEMS and PHASE MEASUREMENTS

This week we will explore a double-sideband (DSB) mixer, use of a mixer as a phase detector, and a digital single-sideband (SSB) mixer. Each student should do everything, but as always we encourage collaboration.

1. BASIC DSB MIXER OPERATION: UPPER AND LOWER SIDEBANDS

For this, use two SRS synthesizer oscillators as inputs to a mixer to explore the spectra and waveforms in the DSB mixing process. The SRS synthesizers work up to 30 MHz. Assign one of the SRS synthesizers to be your "local oscillator" with frequency f_{lo}, and the other your "signal" with frequencies $f_s = f_{lo} \pm \delta f$. Make δf somewhat small compared to f_{lo}, maybe 5% of f_{lo}. For the input level, a good choice is 0 dbm for both synthesizers.

We will want to digitally sample the mixer output and explore both the sum and difference frequencies. As you learned in the Fourier lab, there are extremely important issues regarding sampling rate. The most basic is the Nyquist criterion. For this lab, we also want enough samples per period to give you a reasonable facsimile of the sine wave when you plot it; from this standpoint, it's not unreasonable to sample at twice Nyquist, or even faster. Another issue is the *number of points* you sample, which must be large enough to give you at least a few periods of the slowest sine wave.

From what you know about mixers, what is the fastest sine wave in the output? This, combined with our above comment and the upper limit on our sampling frequency (20 MHz), would determine the upper limit on f_{lo}.

1.1. The Mixer

Combine the two signals in a mixer for the two values for f_s. For the mixer use a Mini-Circuits ZAD-1, which has three BNC connectors (three *ports*) and works well at these frequencies. The ZAD-1, like nearly all mixers, has its ports labeled "R" (the "RF" or "signal"); "L" (the "local oscillator"); and "X" (the "mixing product") or "I" (the "intermediate frequency"). However, as we explain below, these labels are misnomers. They are based on the usual use for a mixer, which is to take two high frequency signals as the inputs to the R and L ports and produce a low frequency difference frequency as the output at the I port.

The ZAD-1 is a balanced mixer, so the "R" and "L" ports are identical, and in particular will not couple to DC or very low frequencies. In contrast, the "I" port is coupled differently and will handle voltages all the way down to, and including, DC. The mixing process functions no matter which two ports are used as inputs. For example, if you are using a mixer to modulate a high frequency (say, a few MHz) with a low frequency (say, a few kHz), you should use the "I" port for the low frequency and either of the other two for the high frequency; take the output from the third port.

We will be wanting to look at the output sum and difference frequencies, so choose the ports appropriately.

1.2. Using the Oscilloscope

Look at the output of the the mixer. You'll have to adjust the trigger level carefully to get a decent trace. Do you understand why the trace looks like it does? Try to determine what frequencies are present from the oscilloscope display. To do so requires careful fiddling with the trigger level and exploring a good range of TIME/DIV.

What do you do with these results? Without your lab notebook, your're sunk! Sketch the traces and annotate the notebook with your interpretive conclusions.

1.3. Digital Sampling

Digitally sample the mixer output for both cases ($f_s = f_{lo} \pm \delta f$). For the analysis, plot the power spectra versus frequency. Explain why the plots look the way they do. In your explanation include the terms "upper sideband" and "lower sideband".

For one of cases, plot the waveform. Does it look like the oscillosceope trace? Then take the Fourier transform (not the power spectrum) of the waveform and remove the sum frequency component by zeroing both the real and imaginary portions (this is "Fourier filtering"). Recreate the signal from the filtered transform by taking the inverse transform (see IDL's documentation for FFT) and plot the signal. Explain what you see.

2. DSB MIXER AS A PHASE DETECTOR

Use a mixer as a phase detector by setting the two synthesizers to the same power level (about 0 dbm) and frequency, i.e. setting $\delta f = 0$. Look at the mixer output on the oscilloscope. Vary the phase of one of the synthesizers. What happens to the mixer output as shown on the oscilloscope trace? What's going on here?

Now measure the mixer output with the HP rack-mounted digital meter in its d.c. mode, reading it with the computer. The d.c. mode is akin to a low pass filter because a the sum frequency, which is a fast sine wave, contains no d.c. Generate a file that contains both the phase and the d.c. voltage and plot (to hand in!) the result. If you're clever, you can write an IDL script to take the data automatically. How does this correspond to the interferometer fringe?

Now displace one of the synthesizer frequencies by some small amount—say, 1 Hz. What happens to the output? This should remind you of the interferometer fringe...

Hand in appropriate plots and discussion.

3. BASIC SSB MIXER OPERATION: UPPER AND LOWER SIDEBANDS

Here we will repeat the DSB experiment except we will use an SSB mixer to separate the sidebands. Here we want to choose our frequencies to be high enough because we will want to filter out the sum frequency using hardware low-pass filters; we have low-pass filters with 5 MHz cutoff. Also, the difference frequency should be $\lesssim$ 1 MHz because the oscilloscope's xy mode doesn't work well above that. Construct the SSB mixer front-end, a quadrature circuit to obtain two mixed outpus, one having phase 90° with respect to the other. Filter out the sum frequency using the hardware low-pass filters on the mixer outputs.

3.1. Using the Oscilloscope

Connect the two mixer outputs to channels 1 and 2 of the oscilloscope and select xy mode. In this mode, the oscilloscope time sweep for the x axis is disabled; instead the x axis position is determined by the channel 1 signal. Similarly, the y axis position is determined by the channel 2 signal. So you are plotting one signal versus the other. For two signals in quadrature, what do you expect to see?[1]

Now set δf very small, about 1 Hz. Watch the trace for the two cases of signal frequency $f_{lo} \pm \delta f$. In your lab report, explain what you did, what you saw, and why it works that way.

3.2. Digital Sampling

Choose δf to be some reasonably high value so you get a bunch of samples without waiting too long. Sample both mixer ouputs simultaneously (in dual channel mode) for the two cases $f_s = f_{lo} \pm \delta f$. Use these two data strings as complex input to the FFT and plot the power spectra. Be sure to include both negative and positive frequencies on your plots. Explain your results.

[1]If you don't see what you expect, it might be because δf is too big or the gains of the x and y channels are not identical,

4. SOME COMMENTARY

Mixers are important because they allow us to shift the frequency of the whole input spectrum by a uniform amount. They do this by multiplying the input signal by the "local oscillator" (l.o.) with frequency f_{lo}; this shifts the frequencies by f_{lo}. In radio reception, this is very important because nearly always our *detectors* work best in a fixed frequency range, but our *signals* come in at many different frequencies. For example, for an AM station playing rock music, the ultimate detector is our ear, which works only at audio frequencies; however, the AM stations transmit at much higher frequency, nearly 1 MHz. A mixer is used to shift the frequencies of the AM station down to the audio region. Such receivers are called *heterodyne* receivers, and this principle is used universally not only in consumer radios, TV's, and cellphones but also in many other applications including radio astronomy.

For example, later in the course we have the 21-cm line at 1420.4 MHz and use an l.o. at 1270.4 MHz, the difference frequency is 150.0 MHz and the sum frequency is 2690.8 MHz. We use a low-pass filter to obliterate the sum frequency, so we are left with a replica of the 21-cm line that is centered at 150.0 MHz instead of 1420.4 MHz. If the Doppler shift from the Earth's orbit (for example) shifts the 21-cm line from 1420.4 to 1420.0 MHz, we change the l.o. frequency to 1270.0 MHz so that the line remains centered at 150 MHz. In this way, our spectrometer can always work at the same frequency, 150 MHz.

Mixers are crucially central to the important radioastronomical technique called interferometry. In this technique, two widely-separated telescopes point at the same source. Usually, the radio source is further from one telescope than the other, which produces a phase difference ϕ. The two received signals from the source are combined in a mixer, which produces a d.c. voltage that is proportional to $\sin\phi$. The phase difference changes with time, which leads to d.c. voltage proportional to $\sin 2\pi f_{loc} t$, where f_{loc} is the local fringe frequency and is equal to $\frac{d\phi}{dt}$. You can also think of of this sinuosidal phase resulting from the differing velocities Δv of the two telescopes with respect to the source, with f_{loc} being the Doppler shift, equal to the observing frequency times $\frac{\Delta v}{c}$.

5. SOME THEORY: THE ORDINARY DOUBLE SIDEBAND (DSB) MIXER

We now turn to the basic theory of the ordinary DSB mixer, which is very straightforward. An ideal mixer multiplies the two input signals together; this multiplication makes the output signal have the sum and difference frequencies. Usually, one of these is eliminated by using a filter.

Suppose for simplicity that the mixer is ideal and that the two input signals are the following: **(1)** the "local oscillator" with voltage equal to unity (for convenience) and frequency ω_0; and **(2)** two "signals" with voltage E_s and frequencies $\omega_{s-} = (\omega_0 - |\delta\omega|)$ and $\omega_{s+} = (\omega_0 + |\delta\omega|)$. We handle the two signal case simultaneously by writing $\omega_{s\pm} = (\omega_0 \pm |\delta\omega|)$.[2]

The mixer outputs in the two cases are MO_- and MO_+; similarly, we write $MO_\pm$. MO is the product of the signal and l.o. signals, and can be expressed in terms of the sum and difference frequencies by the usual trig identity

$$MO_\pm = E_s \underbrace{\cos[\omega_{s\pm} t] \cos[\omega_0 t]}_{product} = \frac{E_s}{2} (\underbrace{\cos[(\omega_{s\pm} - \omega_0)t]}_{diff} + \underbrace{\cos[(\omega_{s\pm} + \omega_0)t]}_{sum}) \tag{1a}$$

Replacing $\omega_{s\pm}$ by $\omega_0 \pm |\delta\omega|$, we get

$$MO_\pm = E_s \underbrace{\cos[(\omega_0 \pm |\delta\omega|)t] \times \cos[\omega_0 t]}_{product} = \frac{E_s}{2} (\underbrace{\cos[\pm|\delta\omega| t]}_{diff} + \underbrace{\cos[(2\omega_0 \pm |\delta\omega|)t]}_{sum}) \; . \tag{1b}$$

Thus, the signal frequency has been shifted to *two* frequencies by an amount equal to the l.o. frequency: downward in the first term (to $\delta\omega$, the difference frequency) and upward in the second term (to $2\omega_0 \pm |\delta\omega|$, the sum frequency). The output contains *both* the difference *and* the sum frequency terms.

Now we insert a low pass filter to eliminate the sum term. We do this because we are observing at some very high frequency, say the 21-cm line at 1.4 GHz, and need to convert to much lower frequencies where our backend equipment works. (In the TV case it's the same: the signals are at hundreds of MHz and the picture processing circuitry operates below 10 MHz). This low-pass filter removes the second term.

So with the low-pass filter we drop the sum frequency term. This leaves the following, in which we add the subscript *cos* to indicate that we began with the signal represented by a cosine:

$$MO_{cos,\pm} = \frac{E_s}{2} \cos[\pm|\delta\omega| t] = \frac{E_s}{2} \cos[|\delta\omega| t] \; . \tag{2a}$$

If we had begun by representing the signal with a sine instead of a cosine (while retaining the cosine for the l.o.) we would have obtained

$$MO_{sin,\pm} = \frac{E_s}{2} \sin[\pm|\delta\omega| t] = \pm \frac{E_s}{2} \sin[|\delta\omega| t] \; . \tag{2b}$$

Four things are important here:

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[2] In real life, e.g. a radio station, the "signal" is speech or music with a broad range of δf. In astronomical life, e.g. the 21-cm line, the "signal" is a Doppler broadened line, which again has a broad range of δf. The system is linear, so signals add without mutual interaction, so the discussion for a single δf also applies to these broad spectra.

1. *The two sidebands—two different input frequencies*—produce the *same final output frequency.* The $+|\delta\omega|$ is known as the "upper sideband" and the $-|\delta\omega|$ as the "lower sideband"; the DSB mixer cannot distinguish between the two.

2. Consider how $|\delta\omega|$ depends on ω_0: for the upper sideband, $\frac{d|\delta\omega|}{d\omega_0} = 1$, while for the lower $d\frac{|\delta\omega|}{d\omega_0} = -1$. We hope that Figure 1 elucidates the situation.

3. A value of E_s for one sideband produces a certain mixer output *power*; the same value of E_s for the other sideband produces the same power. With regard to power, the sidebands are *identical.*

4. But the sidebands are not *completely* identical; see section 7 for a more complete discussion. The sine term changes changes sign for the lower sideband. Consider representing the output voltage in the complex plane. For the upper sideband, which has identical signs for the (cosine, sine) term, the vector rotates counterclockwise. For the lower sideband, it rotates clockwise. Later, this distinction will allow us to construct a *single-sideband* mixer, which provides *two* outputs, one for each sideband.

Figure 1 illustrates these mathematical results. The top panel shows the original RF spectrum. The second panel shows the USB after DSB mixing: it appears at negative and positive frequencies and the spectum is symmetric, meaning that the negative frequencies give exactly the same result as the positive ones. The third panel shows the same for the LSB. You can achieve the rejection of either the LSB or the USB by using an appropriate *bandpass* filter. The fourth panel shows what happens without a bandpass filter: the LSB and USB are inextricably mixed and you get the sum of the power spectra. The bottom panel shows that SSB mixing retains the sideband separation and identity.

6. THE SSB MIXER

The SSB mixer has the capability of distinguishing whether the difference frequency $|\delta\omega|$ is positive or negative—that is, it distinguishes between the two sidebands. The sidebands can, and usually do, contain completely independent signals. The SSB mixer has two outputs, one for each sideband.

These signals really are completely independent. For example, FM broadcasts are usually done in stereo and one could transmit the left and right hand channels in the two sidebands; in fact, for a variety of reasons, it's done differently, with the upper sideband carrying the sum of the left/right channels and the lower sideband the difference.

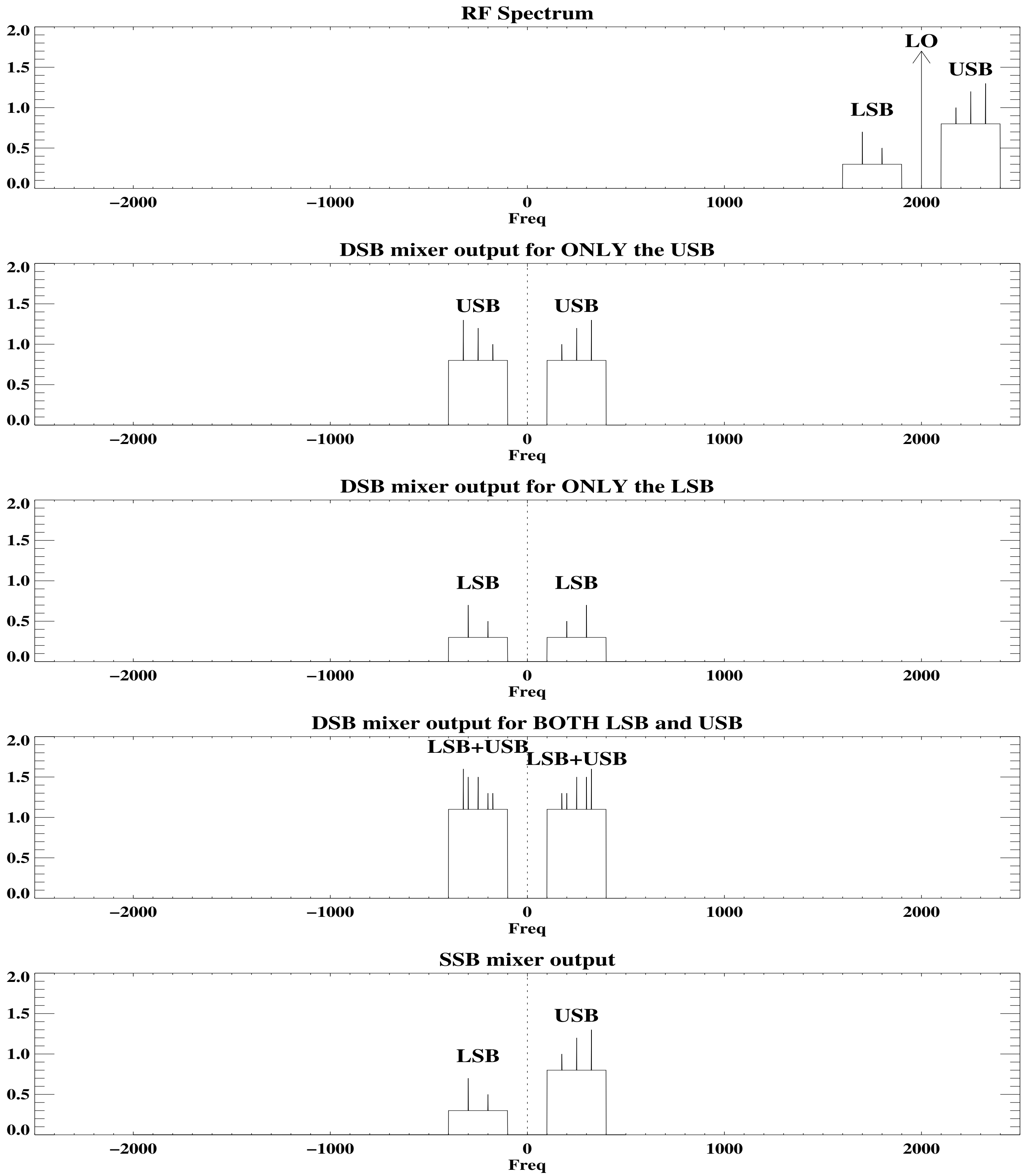

Fig. 1.— Upper and lower sidebands in DSB and SSB mixers for a set of δ-function test signals on top of broad level noise spectra. Top: the RF spectrum. The next two show the USB and LSB *individually* when they undergo the DSB mixing process; panel 4 shows how they both add together. The bottom panel shows the SSB mixer, which keeps them separate.

6.1. A Preliminary: The Quadrature Hybrid

The heart of a SSB mixer is not the mixer. Rather, it's the quadrature hybrid. So before considering SSB mixers, we need to understand hybrids and quadrature hybrids[3].

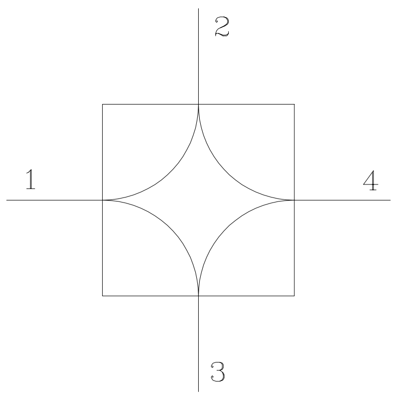

Fig. 2.— Block diagram of a four port hybrid.

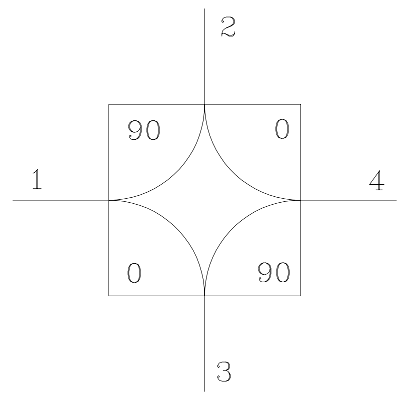

Fig. 3.— Block diagram of a quadrature hybrid.

Figure 2 shows the block diagram of a hybrid. The signal acts just like the diagram shows: a signal entering port 1 will split into equal signals (–3 dB each) and exit through ports 2 and 3. No power is transmitted across the hybrid to port 4. Internally, many hybrids look like center-tapped transforemers; see Hagen's book.

Power is split between the two adjacent ports and nothing is transmitted across it. In this way we think of a hybrid as a *power splitter*. In fact, this is just what power splitters are: a hybrid with the fourth port terminated in a matched load. If you are wondering if it can be used in reverse as a power combiner, the answer is yes. If inputs are made to ports 1 and 4, the sum of the inputs exit through ports 2 and 3. One point: all ports must present matched loads.

Hybrids can be made so that they add a phase angle, either 90° symmetrically in two legs or 180° in one leg. Figure 3 shows a block diagram. A signal incident on port 1 splits and exits through ports 2 and 3, but each signal is *shifted* by the number of degrees specified in each corner of the schematic: the output of port 2 is shifted by 90 degrees with respect to the input while the output of port 3 is not shifted at all.

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[3]For an excellent and thorough discussion of hybrids and their applications, refer to: Hagen, Jon B., "Radio-Frequency Electronics, Circuits and Applications," Cambridge University Press, 1996, pg. 188-200.

6.2. The SSB mixer: block diagram and graphical description

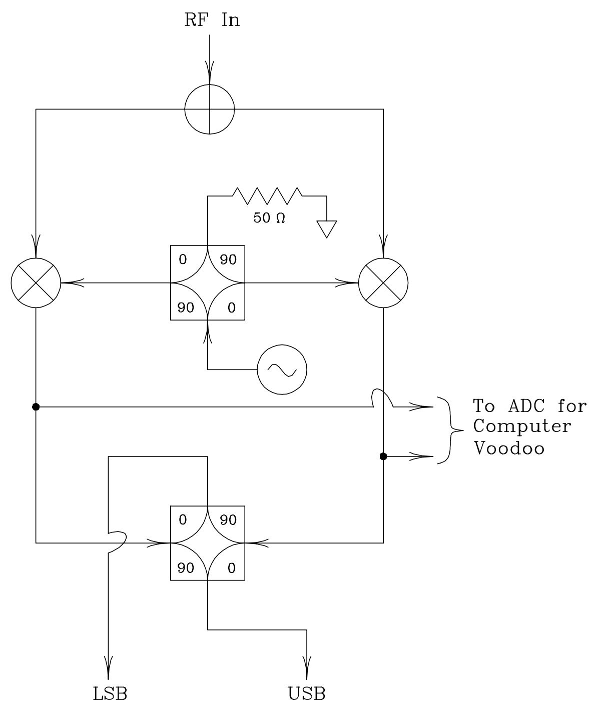

Fig. 4.— Block diagram of the all-hardware SSBM and hardware/software combination SSBM

Figure 4 shows a block diagram of the SSB mixer. The astronomical signal is split by a power splitter into two equal channels. The LO is split by a hybrid, so that the left hand side (LHS) leads the RHS by 90°. These two channels are recombined with the second quadrature hybrid.

Suppose that the signal "RF in" is a cosine wave in the lower sideband, with frequency below the l.o. frequency. The l.o. is a cosine wave on the RHS and a sine wave on the LHS (because of the 90° shift from the hybrid). This makes the two mixer outputs $\propto \sin[\delta\omega t]$ and $\cos[\delta\omega t]$ for the LHS and RHS, respecively. These are shown in Figure 5, with the LHS side $[MO(LHS)_-]$ dashed and the RHS $[MO(RHS)_+]$ solid. These are identical in amplitude: they have to be, because they come from the same original lower-sideband signal. But they are shifted in phase: the dashed curve lags behind the solid one.

Now suppose we shift the *dashed* curve forward by 90°; then it *adds* constructively with the solid curve and gives twice the amplitude. Alternatively, suppose we shift the *solid* one forward by 90°; then the two signals *cancel*.

How do we do this 90° phase shifting? We can do it digitally, by taking its Fourier transform with the two inputs being the real and imiaginary components of the input numbers (which are complex). Or we can do it with another 90° hybrid!

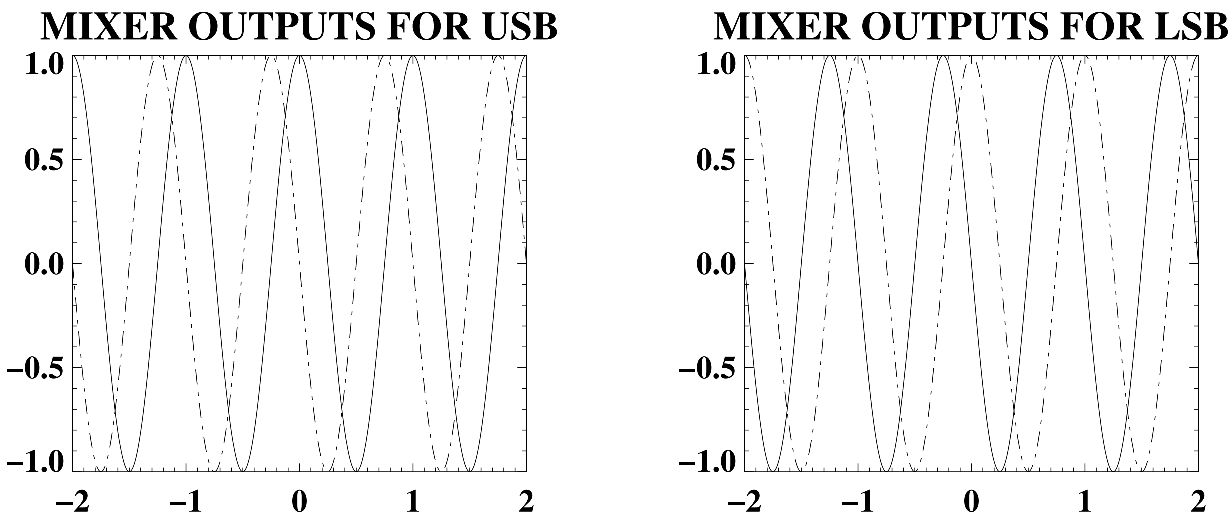

Fig. 5.— Outputs of the first mixers for the two sideband cases. Dashed is the left-hand mixer (LHS), solid is right-hand mixer (RHS).

6.3. The Digital "Computer Voodoo" Method

In Figure 4, we can regard the two independent signals marked "To ADC for Computer Voodoo", as as the real and imaginary components of the signal because, no matter what the input frequency, they differ in phase by 90°. So the signal is a complex number. Above, the signal was a real number, so its Fourier transform had to have Hermitian symmetry—meanang that the power spectrum was symmetric, with negative and positive frequencies being identical. Here, the signal is a complex number, and there is no symmetry. We won't delve into the math here, but the net result is that the Fourier transform keeps the negative and positive frequency components completely separate.

6.4. The Analog method

You don't have to do it digitally. In Figure 4, the second quadrature hybrid at the bottom does exactly this combination of functions! So a lower-sideband signal comes out of the LSB port, and an upper-sideband one comes out of the USB port. The phase combining is linear, so if there exist, simultaneously, different signals in the two sidebands, their separation is maintained at the output.

6.5. Mathematical description

First, we mix the incoming signal with the real and imaginary (cosine and sine) components of the l.o. As in the DSB mixer, we use a low pass filter so we need consider only the difference frequencies. For the left hand side the l.o. has the 90° phase difference so it's a sin instead of a cos and for the Mixer Output (MO) we have

$$MO(LHS)_{\pm} = \frac{E_s}{2} \sin[\pm|\delta\omega|t] \tag{3}$$

so, writing the results separately for the two sidebands as we did asbove, we have

$$MO(LHS)_{-} = \frac{E_s}{2} \sin[-|\delta\omega|t] = -\frac{E_s}{2} \sin[|\delta\omega|t] \tag{4a}$$

$$MO(LHS)_{+} = \frac{E_s}{2} \sin[+|\delta\omega|t] = \frac{E_s}{2} \sin[|\delta\omega|t] \tag{4b}$$

Similarly, for the RHS we have

$$MO(RHS)_{-} = \frac{E_s}{2} \cos[-|\delta\omega|t] = \frac{E_s}{2} \cos[|\delta\omega|t] \tag{5a}$$

$$MO(RHS)_{+} = \frac{E_s}{2} \cos[+|\delta\omega|t] = \frac{E_s}{2} \cos[|\delta\omega|t] \tag{5b}$$

The RHS and LHS outputs have identical frequencies and amplitudes.

But the phase relationships differ for the two sidebands. For the *lower* sideband, the RHS output *lags* the LHS by 90°; for the *upper* sideband, the RHS output *leads* the LHS by 90°. In other words, for the RHS output, lower-sideband signals differ in sign from upper-sideband ones; this is equivalent to differing in phase by 180°.

In the second half of the SSB mixer the signals are recombined in a second 90° hybrid. Referring to Figure 4, the LSB (lower sideband) output is equal to the sum of RHS plus LHS delayed by 90°, conveniently written

$$MO(LSB) = MO(LHS) + MO(RHS)\angle 90^\circ \tag{6a}$$

In contrast, the USB output is

$$MO(USB) = MO(LHS)\angle 90^\circ + MO(RHS) \tag{6b}$$

6.6. Lower sideband output

Now consider the lower sideband output $MO(LSB)$. For the *lower sideband signal* of frequency $(\omega_0 - |\delta\omega|)$, the relevant quantities are $MO(LHS)_-$ and $MO(RHS)_-\angle 90°$. These are just

$$MO(LHS)_- = -\frac{E_s}{2}\sin[|\delta\omega|t] \tag{7a}$$

$$MO(RHS)_-\angle 90° = \frac{E_s}{2}\cos[|\delta\omega|t + 90°] = -\frac{E_s}{2}\sin[|\delta\omega|t] \tag{7b}$$

Adding them gives $MO(LSB)_-$, so we have

$$MO(LSB)_- = MO(LHS)_- + MO(RHS)_-\angle 90° = -E_s\sin[|\delta\omega|t] \tag{8a}$$

If we carry through the same algebra for the *upper sideband signal* of frequency $(\omega_0 + |\delta\omega|)$, we get

$$MO(LSB)_+ = MO(LHS)_+ + MO(RHS)_+\angle 90° = 0 \tag{8b}$$

So we see that the LSB output *excludes the positive frequencies* (upper sideband), as promised.

6.7. Upper sideband output

We can go through the identical steps for the USB output. We then obtain

$$MO(USB)_- = MO(LHS)_-\angle 90° + MO(RHS)_- = 0 \tag{9a}$$

$$MO(USB)_+ = MO(LHS)_+\angle 90° + MO(RHS)_+ = E_s\cos[|\delta\omega|t] \tag{9b}$$

and the USB excludes the lower sideband.

7. AN ASIDE: COMPLEX NOTATION AND POSITIVE/NEGATIVE FREQUENCIES

Remember the relationship between trig functions and complex exponentials:

$$\sin(\omega t) = \frac{1}{2j}[\underbrace{\exp(+j\omega t)}_{pos\ freq} - \underbrace{\exp(-j\omega t)}_{neg\ freq}] \tag{10a}$$

$$\cos(\omega t) = \frac{1}{2}[\underbrace{\exp(+j\omega t)}_{pos\ freq} + \underbrace{\exp(-j\omega t)}_{neg\ freq}] \tag{10b}$$

This means that any sine wave can be considered as a superposition of two frequencies, one positive and one negative; ditto for a cosine wave. We can speak of the "negative frequency branch" and the "positive frequency branch".

What's this "negative frequency" business—is it just a mathematical construction? *NO!!!!* Consider a rotating wheel: cars can go backwards as well as forwards. The wheel rotates $\frac{\omega}{2\pi}$ times per second; one direction is positive frequency, one is negative.

Suppose we rewrite equation 10 in complex form, in which the cosine and sine terms are real and complex. This gives

$$MO_{complex,\pm} = MO_{cos,\pm} + jMO_{sin,\pm} \tag{11a}$$

or, from equation 2

$$MO_{complex,\pm} = \frac{E_s}{2}[\cos(|\delta\omega|t) + j\sin(|\delta\omega|t)] \tag{11b}$$

or the convenient shorthand form

$$MO_{complex,\pm} = \frac{E_s}{2}e^{2\pi j|\delta\omega|t}\ . \tag{11c}$$