CROSSCORRELATION SPECTROPOLARIMETRY IN SINGLE-DISH RADIO ASTRONOMY

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ABSTRACT

Modern digital correlators permit the simultaneous measurement of all four Stokes parameters using auto and crosscorrelation. We briefly outline the fundamental requirements and some practical details of performing such measurements and refer to two additional papers that provide overview and cover calibration issues in detail.

Subject headings: polarization — instrumentation: polarimeters — techniques: polarimetric

1. INTRODUCTION TO CROSSCORRELATION SPECTROPOLARIMETRY

Modern digital correlators permit complete, simultaneous spectropolarimetric measurements by providing the frequency spectra of all four Stokes parameters using auto and crosscorrelation. This paper is a very brief introduction to two additional papers (Heiles et al 2001a, 2001b) that provide detailed explanations of the calibration issues of such measurements. One of these deals with the measurement and calibration of instrumental effects on Stokes parameters using the 4×4 Mueller matrix. The other deals with the more general issue of parameterizing the telescope beam and first sidelobe, for not only the usual Stokes I but also the other three Stokes parameters; beam polarization effects can lead to serious instrumental effects when measuring extended emission.

Our work is based on experience with systems at three telescopes: the Spectral Processor at the NRAO 140 foot telescope[1], the interim correlator at the Arecibo telescope[2], and the large digital correlator at the NRAO 12-m telescope. We learned about instrumental effects that can affect the astronomical measurements unless they are properly calibrated and equipment details that need to be accounted for in the calibration.

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[1]The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc.

[2]The Arecibo Observatory is part of the National Astronomy and Ionosphere Center, which is operated by Cornell University under a cooperative agreement with the National Science Foundation

In the present short paper we briefly outline the the technique and cover a few practical details of performing such measurements. Our purpose here is not to present a complete overview or to reiterate the many important details discussed at length in the other papers. Rather, it is to present the briefest of brief introductions and, in addition, to cover several practical details for the specialist. The basic reference for our work is the excellent book on astronomical polarization by Tinbergen (1996). A more mathematical and fundamental reference is Hamaker, Bregman, and Sault (1996), which the theoretically-inclined reader will find of interest.

The four Stokes parameters define the complete state of polarization of electromagnetic radiation. They are most simply thought of as linear combinations of orthogonal polarizations. Stokes I, the total intensity, is the sum of any two orthogonal polarizations. Stokes Q and U define the linear polarization: $Q = i_0 - i_{90}$ and $U = i_{45} - i_{-45}$, where i is the power measured in a particular polarization and the numerical subscripts define the position angle of the received linear polarization. Stokes V defines the circular polarization: $V = i_{LCP} - i_{RCP}$. Here we follow the IEEE definition for circular polarization; LCP means left-hand-circular polarization. LCP is generated by transmitting with a left-handed helix, so from the transmitter the E vector appears to rotate anticlockwise. From the receiver, LCP appears to rotate clockwise. Generally in this paper we follow Tinbergen (1996), whose definition of LCP is the same as ours; however, his definition of V (see his page 61) is opposite ours and, also, the conventional one used by radio astronomers.

More generally, the Stokes parameters can be written as time-average products of the electric fields. In a radio astronomy feed these electric fields are converted to voltages, which are equivalent. Let us focus on the specific case of two orthogonal linear polarizations that provide voltages (E_X, E_Y). Here the directions (X, Y) specify the coordinate system with respect to which PA_{src}, the polarization position angle of an astronomical source, is measured; by astronomical convention, PA_{src} is measured with respect to the North Celestial Pole, increasing towards the East, so X points North and Y points East.

For a monochromatic signal, the voltages (E_X, E_Y) vary sinusoidally with time and can be written in complex notation, for example $E_X = E_{X0}e^{i(2\pi ft+\psi_x)}$, where f is the observing frequency and ψ is a phase angle. An arbitrary signal is a sum of such monochromatic signals and is also complex. We then have

$$I = E_X\overline{E_X} + E_Y\overline{E_Y} \tag{1a}$$

$$Q = E_X\overline{E_X} - E_Y\overline{E_Y} \tag{1b}$$

$$U = E_X\overline{E_Y} + \overline{E_X}E_Y \tag{1c}$$

$$iV = E_X\overline{E_Y} - \overline{E_X}E_Y \ . \tag{1d}$$

where the bars indicate complex conjugate and it is understood that the products are averaged over time. These time average products can be obtained by suitable equipment that multiplies the voltages. In practice the products are performed digitally, with the real and imaginary portions being obtained by mixing the incoming signal with a local oscillator having 0° and 90° phase shifts.

To obtain spectral information, there are two almost equivalent techniques. More usual in single-dish work is the XF technique: one calculates time-average correlation functions digitally (the X part) and takes the Fourier transform (the F part). The alternative is the FX technique, in which one calculates Fourier transforms (F), multiplies by the complex conjugate (X), and averages the resulting spectra. According to the convolution theorem, these produce identical results. However, it is not generally appreciated that the convolution theorem applies only if the sampling time is infinite. For real data with a finite sampling time, the off-frequency spurious response, sometimes called "spectral leakage", differs for the two techniques. For example, consider the example of a monochromatic signal of frequency f_0. In the XF case, the spectral leakage has the form $\frac{\sin x}{x}$, where $x \propto (f - f_0)$, and the power spectrum can be negative. In contrast, with FX processing the power spectrum is the product of a Fourier transform with its own complex conjugate, and is therefore everywhere positive. For the XF technique the spectral leakage can be reduced by appropriate convolution of the power spectrum, which is equivalent to weighting the time window, e.g. with Hanning weighting. However, with the FX technique convolution doesn't work and the window weighting must be done on the Fourier transformed voltages before the multiplication.

The Green Bank Spectral Processor uses the FX technique; its output is time-average products of Fourier transforms. In equations 1, one should replace the voltage products by these products of Fourier transforms. For example, $E_X\overline{E_X}$ is replaced by $FT(E_X)\overline{FT(E_X)}$ and $E_X\overline{E_Y}$ is replaced by $FT(E_X)\overline{FT(E_Y)}$, where FT means Fourier transform. The FT's are complex, so while the self products [like $FT(E_X)\overline{FT(E_X)}$] are always real, the cross products [like $FT(E_X)\overline{FT(E_Y)}$] are complex.

Standard digital correlators, like the Arecibo and NRAO correlators, use the XF technique; the outputs are time-average correlation functions. In equations 1, one should replace the voltage products by Fourier transforms of these correlation functions. For a single signal like E_X, the correlation function is the autocorrelation function. Autocorrelation functions are symmetric about zero time lag and therefore the Fourier transforms are real; these produce Stokes I and Q above. In contrast, for the two signals of orthogonal polarizations E_X and E_Y, the correlation function is the crosscorrelation function. The cross correlation function has arbitrary symmetry so its Fourier transform is complex; the real part is U and the imaginary one V. These matters are discussed at length in books on interferometry, e.g. Thompson, Moran, & Swenson (1994).

Equations 1 assume that the voltages are properly calibrated for polarization. "Calibration for polarization" means, specifically, that the relative gains and phases of the two polarization channels are measured and the differences corrected for. Heiles et al (2001a) explains these calibration matters in detail. An additional aspect of calibration is the establishment of an accurate scale of antenna temperature or flux density; this aspect is beyond the scope of the current discussion, which focuses on polarization calibration.

Two quantities, the relative gain and phase of the two polarization channels, must be calibrated. In principle, a good way to do this is with an astronomical source having known linear polarization. However, in practice this is not sufficient because the phase is ambiguous by 180° (see §2.1) and, moreover, the phase delay can change with telescope position, e.g. when the i.f. cables are mechanically strained at the telescope bearings. Therefore, a secondary calibration standard that is attached to the telescope, one that can be used quickly without moving the telescope, is required. This standard is best realized by injecting a correlated noise source into the two feed outputs or, alternatively, by radiating a noise source into the feed with a polarization that provides equal amplitudes in the feed outputs. Under some circumstances it is desirable to inject noise separately into the two inputs to simulate uncorrelated noise; these matters are discussed in detail by Heiles and Fisher (1999).

In the following sections we describe several practical details associated with the technique of single-dish crosscorrelation spectropolarimetry. These details go beyond the calibration techniques conventionally required for single-dish radio astronomy, such as are discussed by Rohlfs and Wilson (1999), for example. Some of these details are familiar to practitioners of interferometric crosscorrelation spectropolarimetry; however, we suspect that persons who concentrate on single-dish techniques may find them unfamiliar (as we did).

2. THE RELATIVE PHASE ψ

We define ψ to be the relative phase between the two polarization channels that enter the correlator. It consists of two parts, the source contribution ψ_{src} and the instrumental one ψ_{sys}. Thus,

$$\psi = \psi_{src} + \psi_{sys} \ . \tag{2}$$

$\psi_{src} = 0°$ or $180°$ for a linearly polarized source and $\pm 90°$ for a circularly polarized one. ψ_{sys} is produced by several effects as discussed below.

2.1. TWO COMMON MISCONCEPTIONS CONCERNING ψ_{src}

Suppose for the moment that $\psi_{sys} = 0°$ and, further, that the feed has orthogonal linear polarizations oriented at position angles $(0°, 90°)$. A common misconception is that the observed ψ from a linearly-polarized source depends on the position angle so that, for example, it changes smoothly with the parallactic angle as the source is tracked with an alt-az telescope. Another is that the observed ψ is independent of position angle. Neither is the case.

The first misconception arises because one erroneously imagines that the relative *amplitude* of E_X and E_Y determine ψ; this amplitude ratio changes with position angle. However, ψ is the *phase* difference between E_X and E_Y. If U is positive, this phase is $\psi = 0°$; if U is negative, $\psi = 180°$. U changes sign when the position angle crosses the boundaries 0° and 90° (recall that position angle is modulo 180°, so each crossing occurs twice in a full rotation).

It is important to remember that the relative phase of a linearly-polarized source is bimodal, with the two values 0° and 180°. It means that without keeping track of the position angle, one cannot rely on an astronomical source to calibrate the instrumental relative phase because of the 180° ambiguity.

2.2. THE EFFECT OF AMPLIFIERS AND SIDEBAND ON ψ_{sys}

One needs to adjust the signal level entering the correlator to its optimum level. This is accomplished with a combination of amplifiers and attenuators, and often this combination must be changed in response to the source intensity. Many types of amplifier introduce 180° phase shifts. If one changes the number of amplifiers in the chain, then the phase can jump by multiplies of 180°; this can be disconcerting. Of course, when switching in attenuators and amplifiers, the path lengths usually change, and this also produces phase changes.

In a similar vein, we found that most of the contribution to ψ_{sys} comes from the i.f. cables. However, we always refer ψ to the r.f. frequency. For that part of ψ_{sys} contributed at any i.f. stage, the sign of its ψ depends on whether that stage processes the upper or lower sideband.

2.3. A PRACTICAL DETAIL IN PHASE CALIBRATION: A ROBUST LEAST SQUARE FIT OF THE PHASE VERSUS FREQUENCY

The relative phase delay caused by the system, ψ_{sys}, can have several causes, but in our experience the most important is differences in cable lengths between the two channels. These differences can occur at r.f. (before the first mixer) and i.f. (after the first mixer). For most telescopes the latter dominates, probably because there is usually a long cable run from the feed to the control room. Not always, however: at the NRAO 12-m telescope, the feeds for the two polarization channels are mounted separately so that the incoming signals have much different path lengths; moreover, the mountings are mechanically soft so that this path length changes with telescope position.

The difference in cable length produces a linear phase difference with frequency given by

$$\frac{d\psi_{sys}}{df} = \frac{2\pi \Delta L}{c} \ , \tag{3}$$

where ΔL is the difference in electrical length and ψ_{sys} the instrumental phase difference between the two channels. For example, at Arecibo we find $\frac{d\psi_{sys}}{df} \sim 0.1$ rad MHz^{-1}, which corresponds to a length difference of ~ 5 m, most of which probably occurs along the pair of ~ 500 m optical fibers that carry the two channels from the feed to the control room.

The quantity $\frac{\Delta L}{c}$ is just the time difference between the two signal paths. This suggests that one could obtain $\frac{d\psi_{sys}}{df}$ by injecting correlated noise at the front end and locating the time offset from zero delay of the peak of the crosscorrelation function. However, this doesn't work in practice, for two reasons. First, the time delay is usually a tiny fraction of the sampling time. For example, at Arecibo the time delay arising from the 5 meter cable length difference is of order 0.01 microsec, while often our observing bandwidth is about 1 MHz so the sampling time is of order 1 microsec. Thus the time delay is about 1% of the sampling interval. Locating a peak to 1% of the sampling interval requires a difficult, ill-defined nonlinear least squares fit. Second, the two versions of signal come through entirely different signal paths, namely the two polarization channels, and typical single dish electronics are usually not engineered to optimize crosscorrelation so that the filter shapes in the two paths are not very well matched. This can make the crosscorrelation function rather complicated and even obliterate the expected well-defined single peak.

In contrast, the phase slope is always very well defined and, moreover, it is precisely the quantity required for calibrating the polarization characteristics. One measures this by injecting a correlated noise source ("cal") at the front end and fitting the linear frequency dependence of the phase difference.

Such fits are best done using the least-squares technique. One might be tempted to make a linear polynomial fit of ψ to frequency. However, this is much more difficult than it seems because ψ suffers sudden wraparound jumps of 2π when it crosses the boundaries $-\pi$ or π. Moreover, ψ has noise, and this makes the locations of these jumps difficult to determine. One can surmount this difficulty by various *ad hoc* subterfuges, but there is a better way.

First, realize that we define the phase ψ by calculating

$$\psi = \tan^{-1}\left[\frac{\mathrm{Im}(E_X E_Y)}{\mathrm{Re}(E_X E_Y)}\right] \ , \tag{4}$$

which means that $\sin\psi = \mathrm{Im}(E_X E_Y)$ and $\cos\psi = \mathrm{Re}(E_X E_Y)$. We write

$$\psi = A + Bf \tag{5}$$

where f is the r.f. frequency and (A, B) are the constants we need to determine. Write

$$\sin(A + Bf) = SA\cos(Bf) + CA\sin(Bf) = \mathrm{Im}(E_X E_Y)\ , \tag{6a}$$

$$\cos(A + Bf) = CA\cos(Bf) - SA\sin(Bf) = \mathrm{Re}(E_X E_Y) \tag{6b}$$

where $(CA, SA) = (\cos A, \sin A)$. Now determine (CA, SA, B) using least squares. A very robust procedure is to use a reasonably accurate guess for B and simultaneously fit for (CA, SA) in equation 6; this is a straightforward linear least squares fit. Then use the results as input estimates in a full three-parameter nonlinear least squares fit.

3. A PRACTICAL DETAIL IN FOURIER TRANSFORMING CROSSCORRELATION FUNCTIONS

One generally uses the FFT algorithm when Fourier transforming correlation functions, with the number of channels equal to an integral power of 2. Some care is required in taking such Fourier transforms of correlation functions.

Digital correlators for single dishes are usually engineered to provide positive time lags. However, crosscorrelation requires both positive and negative lags. One obtains the full crosscorrelation function of (E_X, E_Y) by measuring *two* correlation functions with positive lags, one delaying E_Y and the other E_X. These must be combined before taking the Fourier transform.

The process of combining the two correlation functions to produce a crosscorrelation function (and that of symmetrizing an autocorrelation function) runs into a particular practical detail. The two halves of a crosscorrelation function each have N channels, with time lags running from $0 \rightarrow (N-1)\tau$ and $0 \rightarrow -(N-1)\tau$, because correlators are always engineered so that both halves measure the zero lag product. (Here τ is the sampling time interval). Thus the total number of independent channels is not $2N$, but rather $2N-1$. However, the FFT algorithm requires $2N$ input numbers.

We have a "missing channel". This missing channel has a time lag equal $N\tau$, and also equal to $-N\tau$: from the fundamental assumptions inherent in digital Fourier transforms, the correlation values for the $\pm N\tau$ must be equal. One must set the unknown correlation value for this channel to a reasonable number. The proper choice for this number is important only insofar as it should produce no discernible impact on the derived power spectrum.

Generally, correlation functions for random noise tend towards zero at large lags. The unknown channel has a large lag, so one might be tempted to set its value to zero. This is the wrong choice! There are two reasons: one, the signal may have some low-frequency components that make its correlation at large lags nonzero; two, the digital correlator may have a d.c. offset. In either case, setting the missing channel equal to zero produces a spike in the correlation function, which is reflected in the power spectrum as a channel-to-channel oscillation. The proper choice for the missing channel is the average of the two values for $\pm(N-1)\tau$, because this produces no extraneous effects in the derived power spectrum.

Often one weights the time window to reduce spectral leakage, as mentioned above. Most weighting schemes, for example Hanning weighting, assign zero weight to the missing channel. With such weightings, the missing channel doesn't matter. Weighting is often desirable, particularly when there is interference, to reduce the spectral ringing.

4. SUMMARY

We have provided a brief introduction to the technique of digital spectropolarimetry in radio astronomy, together with fundamental references. We also include some comments and details based on experience that should help a specialist to get started with this technique.

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